Evolutionary Multiobjective Optimization Methods for the Shape Design of Industrial Electromagnetic Devices

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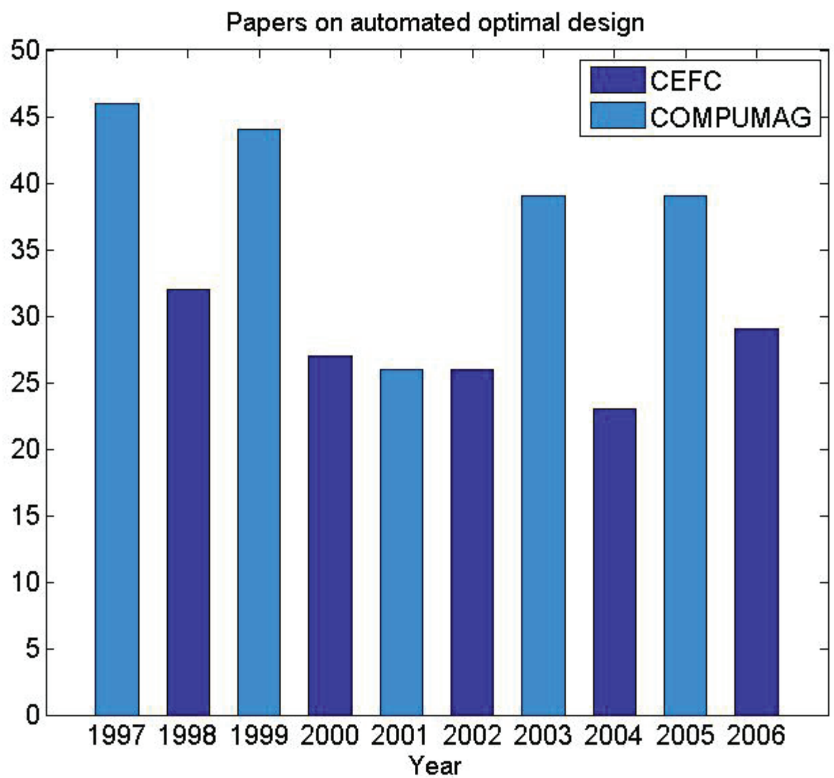
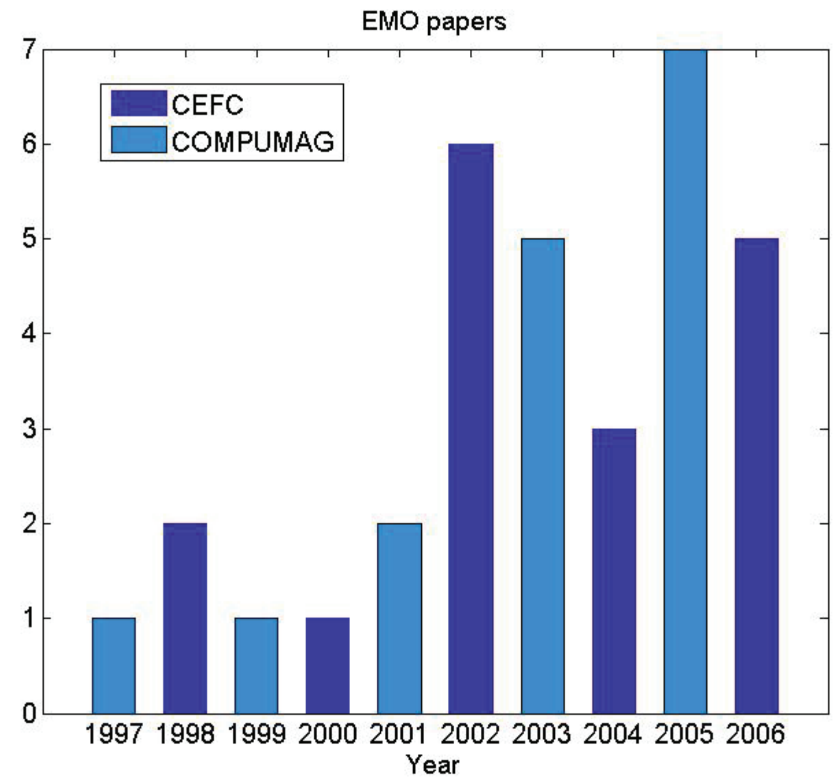
INTRODUCTION

Evolutionary Multiobjective Optimization (EMO) opened a new research field in electromagnetism





Main applications

low frequency: electromechanics, magnets

high frequency: antenna design

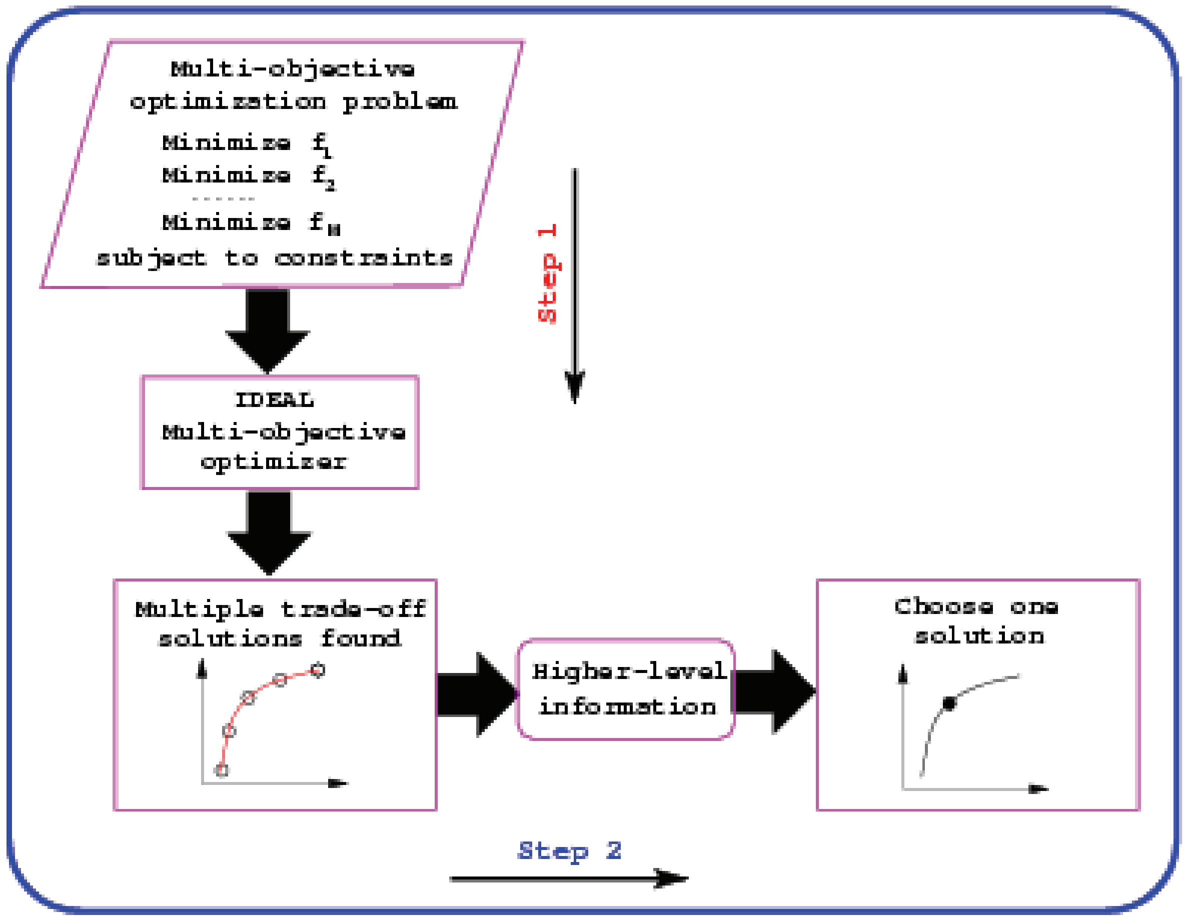
IDEAL MULTIOBJECTIVE OPTIMIZATION

Step 1

Find a set of Pareto-optimal solutions

Step 2

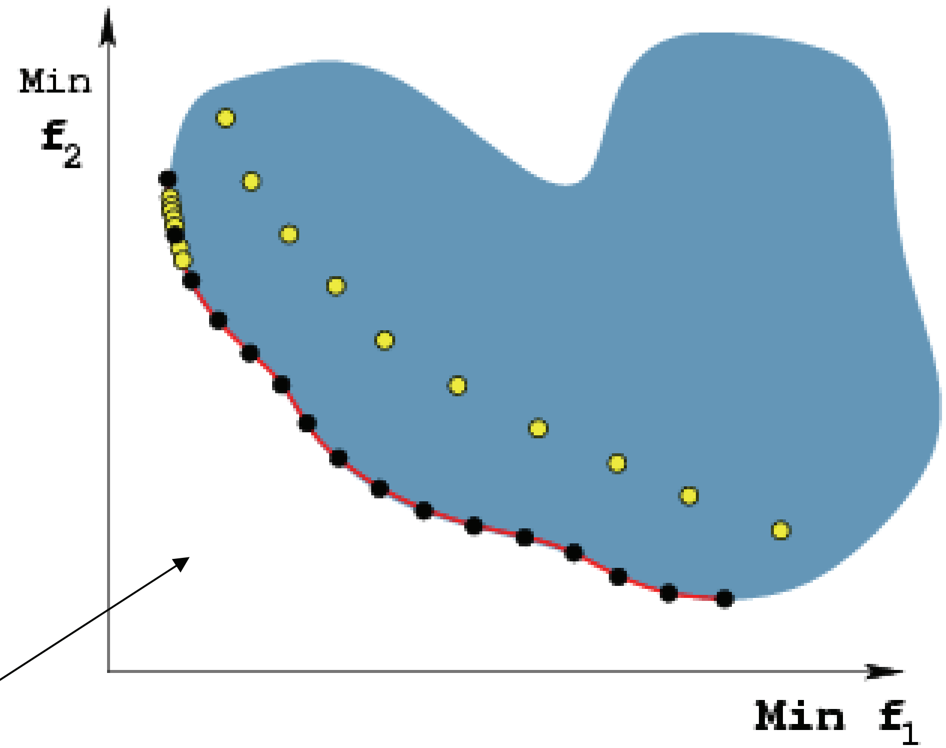
Choose a solution from the set



The Pareto-optimal front is the boundary of the feasible search region in the objective space (f_1 , f_2 ,…, f_m)

Two goals in ideal multiobjective optimization

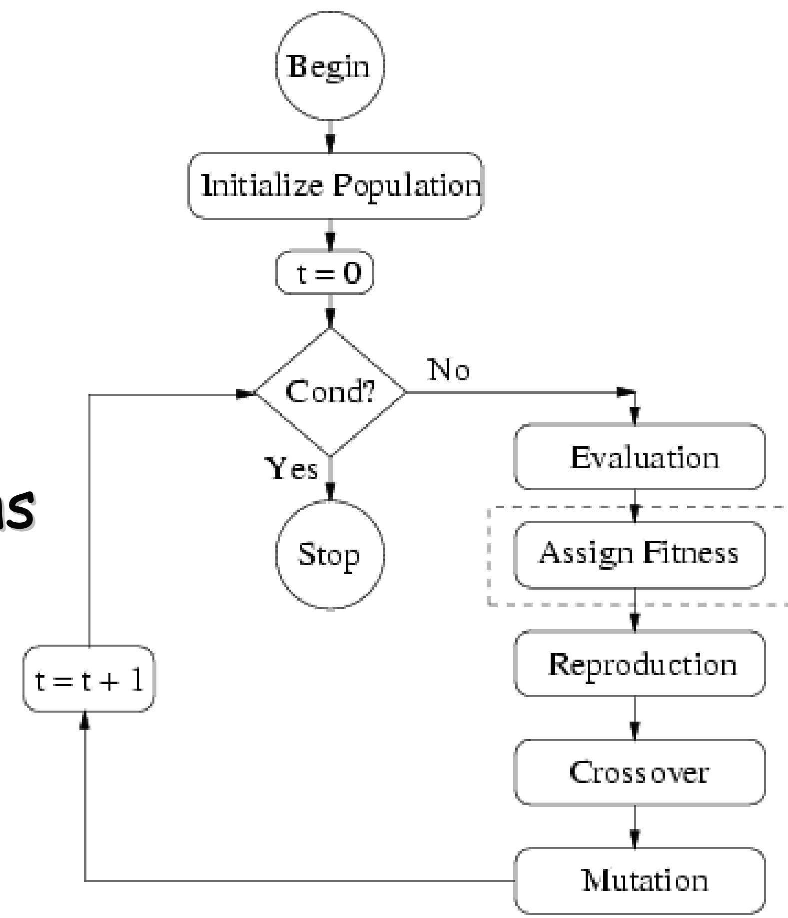
- Converge to the Pareto-optimal front
- Keep as diverse a distribution as possible



unfeasible region

Population-based approach
What to change in a basic GA ?

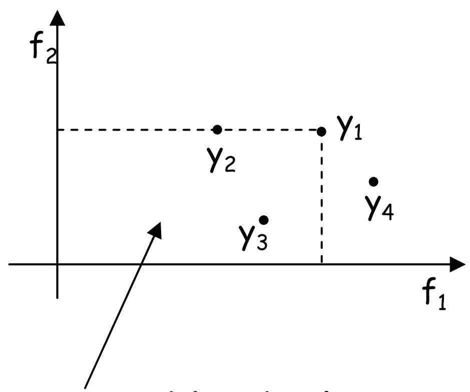
- Modify the fitness computation
- Emphasize non-dominated solutions for convergence
- Emphasize less-crowded solutions for diversity



Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

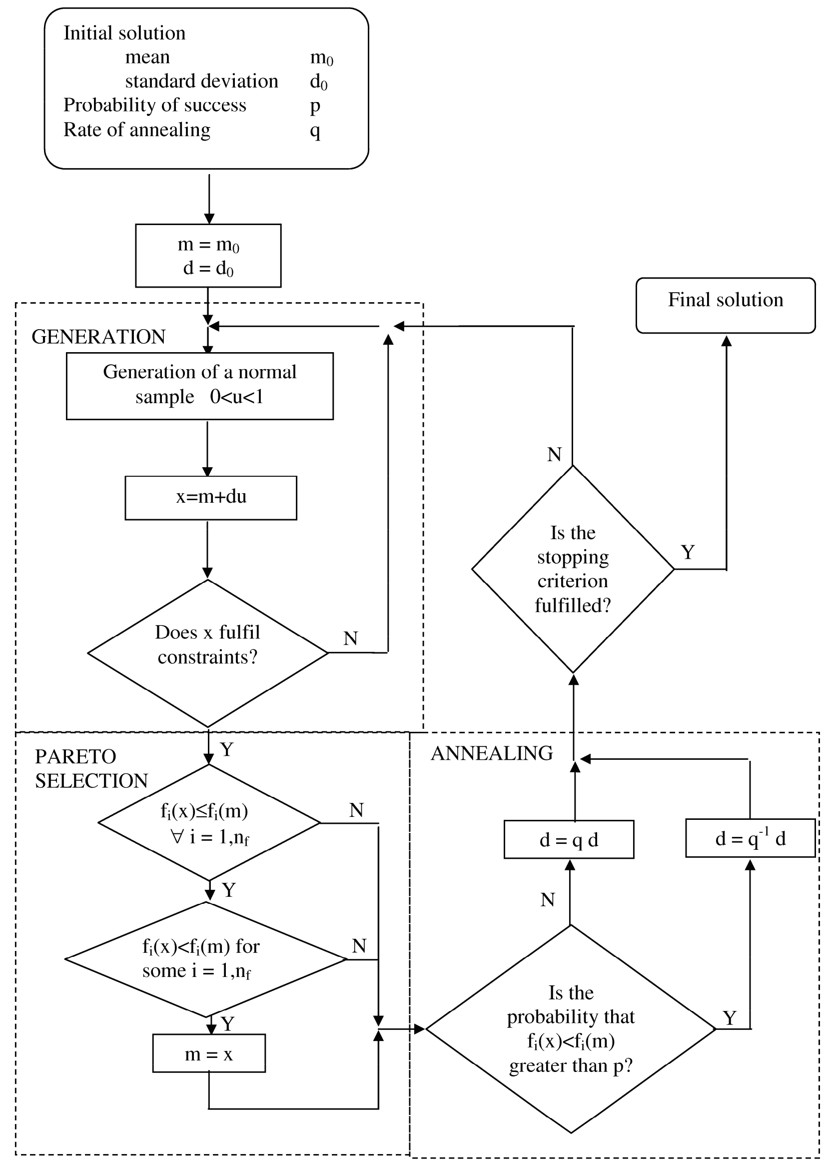
Individual-based approach
What to change in a basic ESTRA ?

Modify the acceptance criterion of the offspring

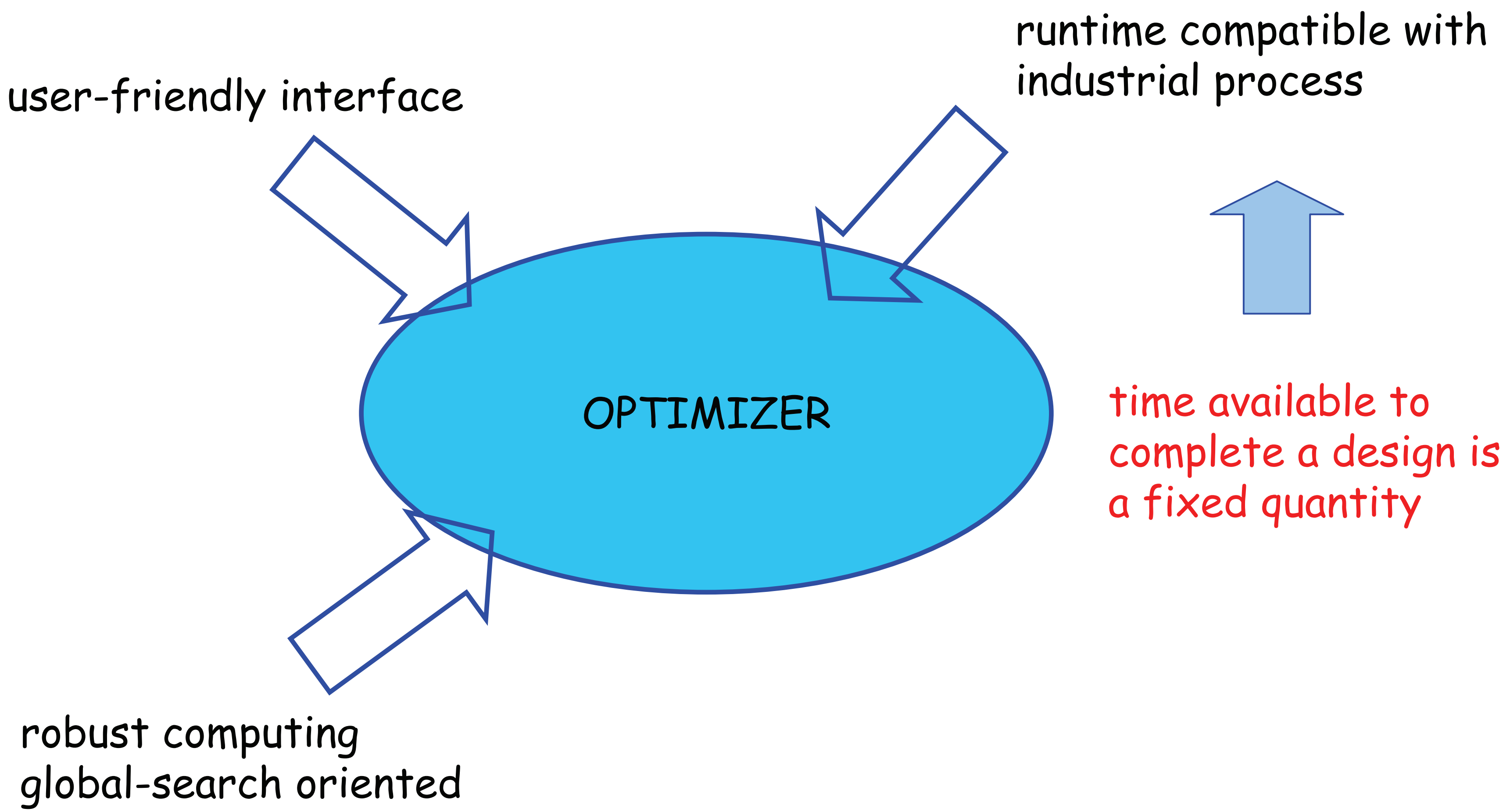


(y_2 , y_3) are accepted,
y_4 is rejected

Multiobjective Evolution Strategy (MOESTRA)



INDUSTRIAL ELECTROMAGNETIC DESIGN



SPEED REQUIREMENTS

Implement software with highest efficiency (including the link to FEM code)

Give the user the freedom to choose a compromise between accuracy and computing time

→ Non-standard stopping criteria

USER INTERFACE REQUIREMENTS

Users should be guided to specify objectives and constraints without having advanced programming and mathematics knowledge

Practical methods to solve EMO in electromagnetism

Two main streams can be observed

- use approximation techniques
- identify a surrogate model of objectives and constraints

then

- use an evolutionary algorithm to optimize

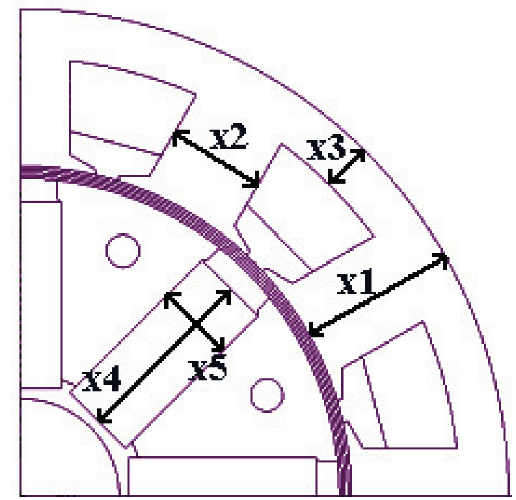
- preserve the use of FE models to solve the direct problem, but
- reduce the solution time of field analysis
- use non-standard stopping criteria for the evolutionary algorithm

⇩ well suited for an industrial R&D centre

CASE STUDY

Permanent-magnet generator for automotive applications.

A very similar device was used as the alternator on board of fast cars for sport competitions.



Design problem: identify the shape of the device such that

- power loss in copper windings $f_1(x) = \int_{\Omega_1(x)} \rho [J(x)]^2 \, d\Omega$
- power loss in the iron core $f_2(x) = \int_{\Omega_2(x)} p(B(x)) \, d\Omega$

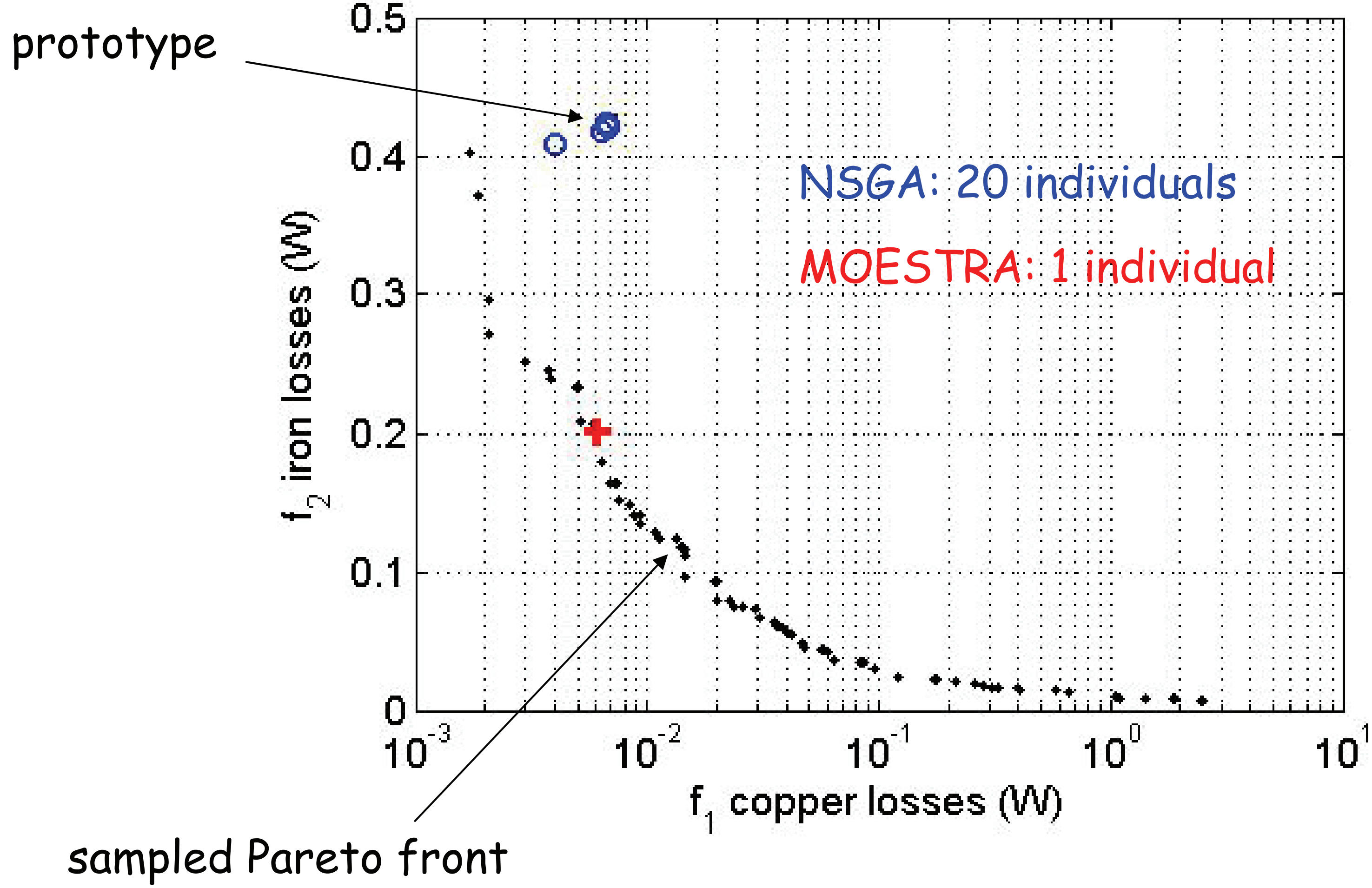
Ω_1 copper volume
Ω_2 iron volume

are minimum.

Constraint : load 500 W, no-load peak voltage 50 V , speed 9,000 rpm

NSGA vs MOESTRA comparison

(prescribed runtime was 6,300 s)



P. Di Barba and M. E. Mognaschi, "Industrial Design with Multiple Criteria: Shape Optimization of a Permanent-Magnet Generator", *CEFC 2008*, Athens

A possible solver of EMO problems: scale-varying evolutionary search

The adaption rate $0<\lambda<1$ of the FE mesh is ruled by the annealing operator of a basic evolution strategy.

A low-cost mesh is generated when a large search radius is taken on and, conversely, a finer mesh is generated when a small region is investigated.

$$\lambda(k) = \lambda_{min}\left(1 - \frac{m(k)}{n}\right) + \lambda_{max}\frac{m(k)}{n}$$

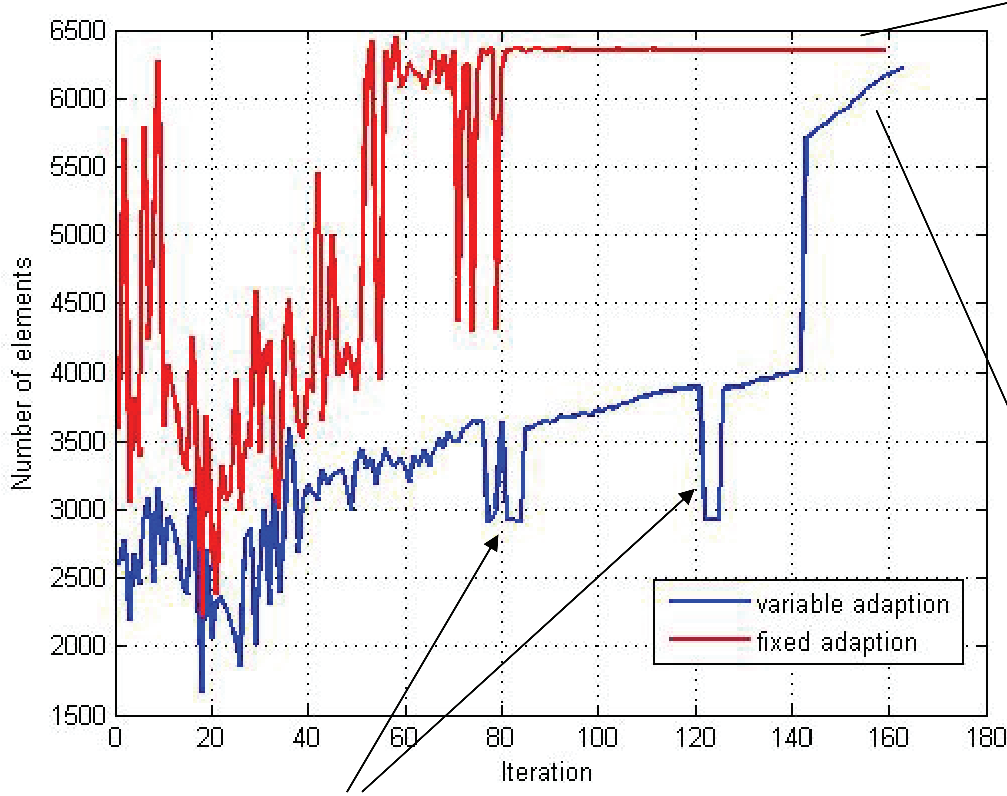
$$n = \frac{\log d_f - \log d_0}{\log q}$$

$$m(k) = \frac{\log d(k) - \log d_0}{\log q}$$

d_0 initial search tolerance
d_f final search tolerance
k iteration index
q annealing rate

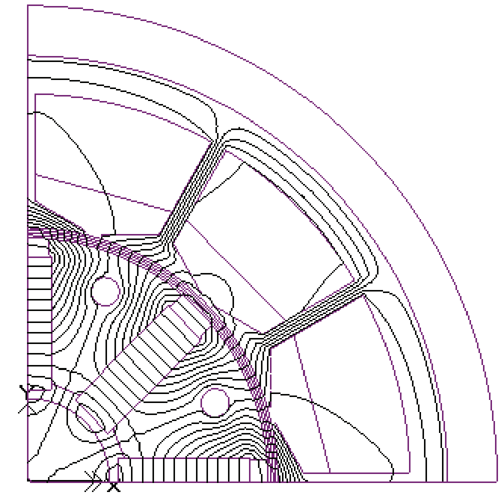
CASE STUDY

Stopping criterion: search tolerance < 10^{-6}



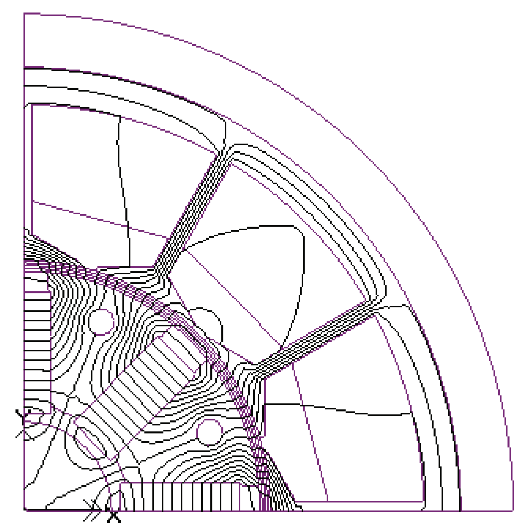
larger search radius

Fixed vs variable adaption



x=[10.33, 1.01, 2.30, 7.99, 2.87] mm

f=[4.55, 160] mW

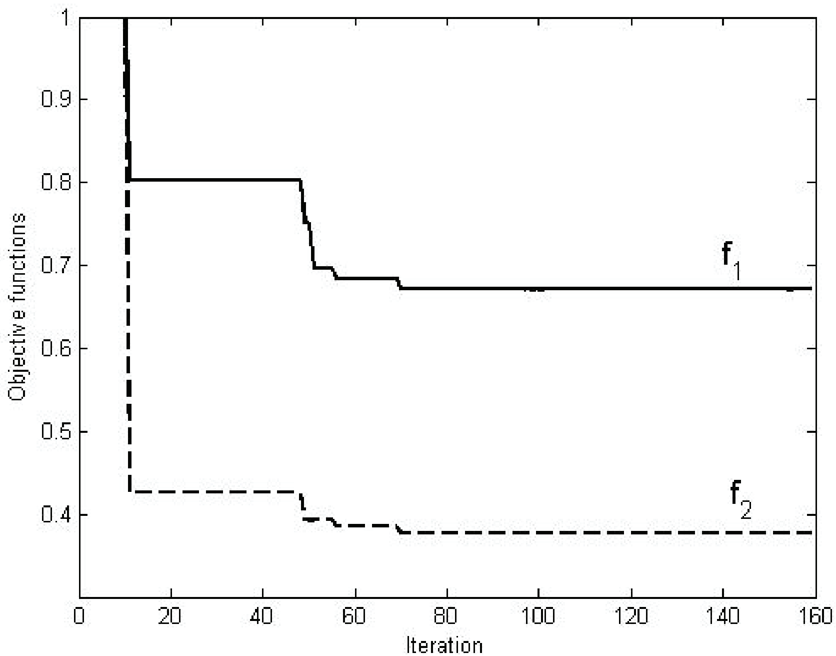


x=[10.87, 0.97, 2.11, 6.83, 3.05] mm

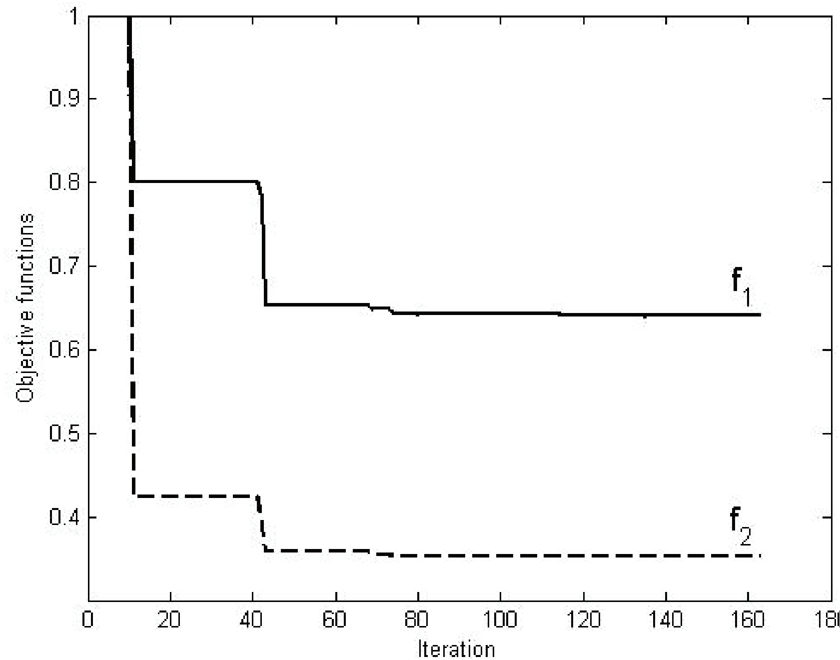
f=[4.36, 150] mW

CASE STUDY

Objective function history



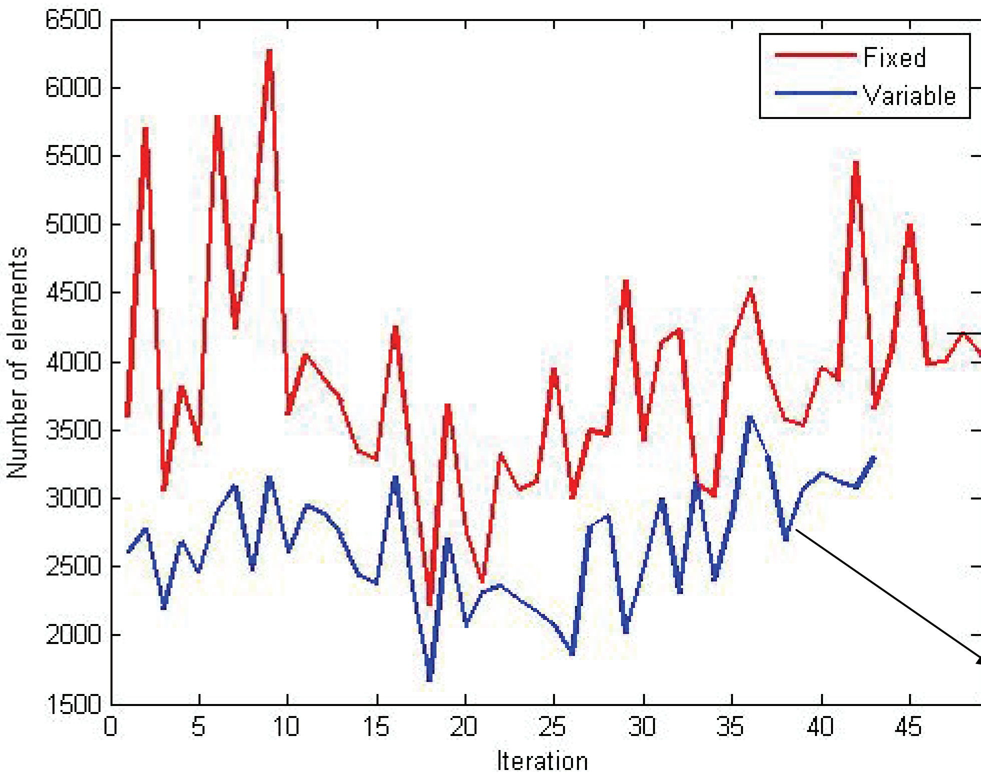
Fixed adaption



Variable adaption

CASE STUDY

User-defined accuracy: the optimization stops when $\rho_i(k) \equiv \frac{f_{ki}}{f_{0i}} \leq \eta\ ,\ i=1,2\ ,\ k \geq 0$

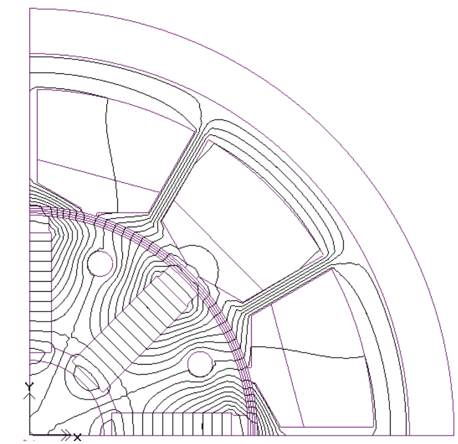


Prescribed η = 0.75

x=[10.18, 1.06, 2.38, 8.03, 2.95] mm

ρ_1=0.75, ρ_2=0.4

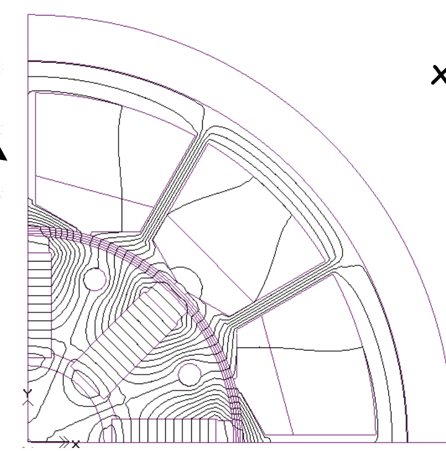
k=49



x=[10.81, 0.99, 2.09, 6.84, 3.07] mm

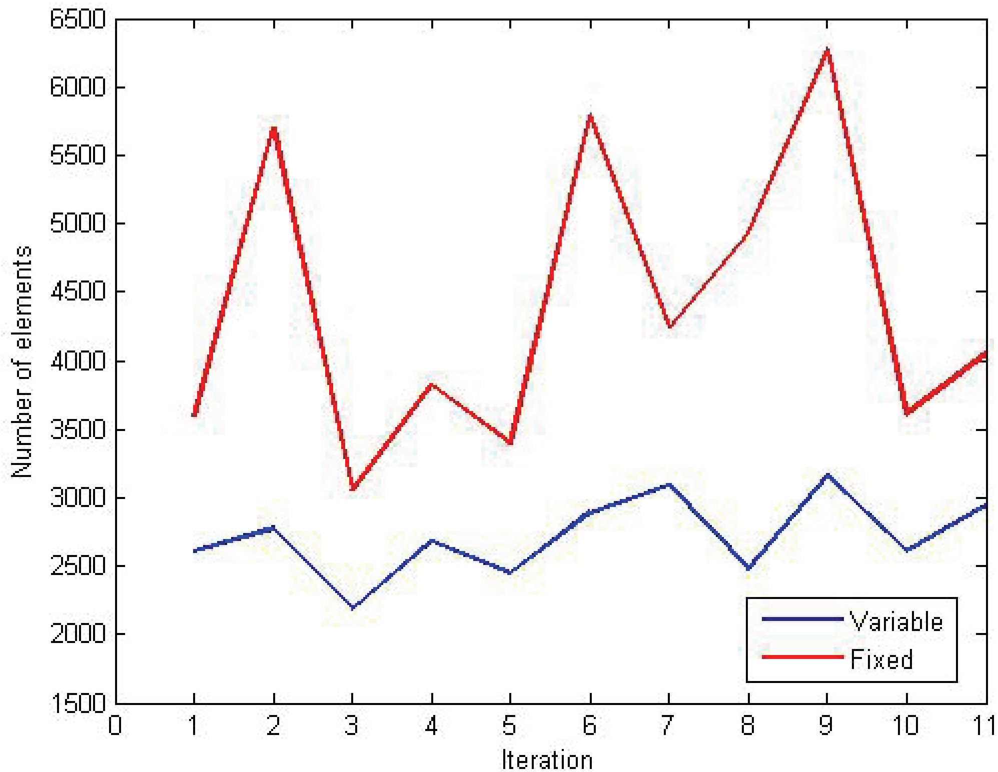
ρ_1=0.65, ρ_2=0.36

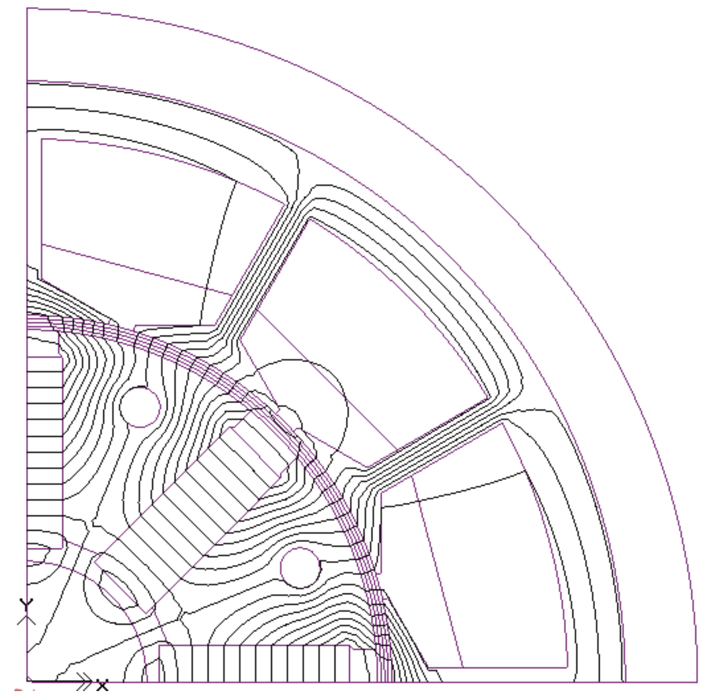
k=43



CASE STUDY

User-defined time: the optimization stops after e.g. 1 hour





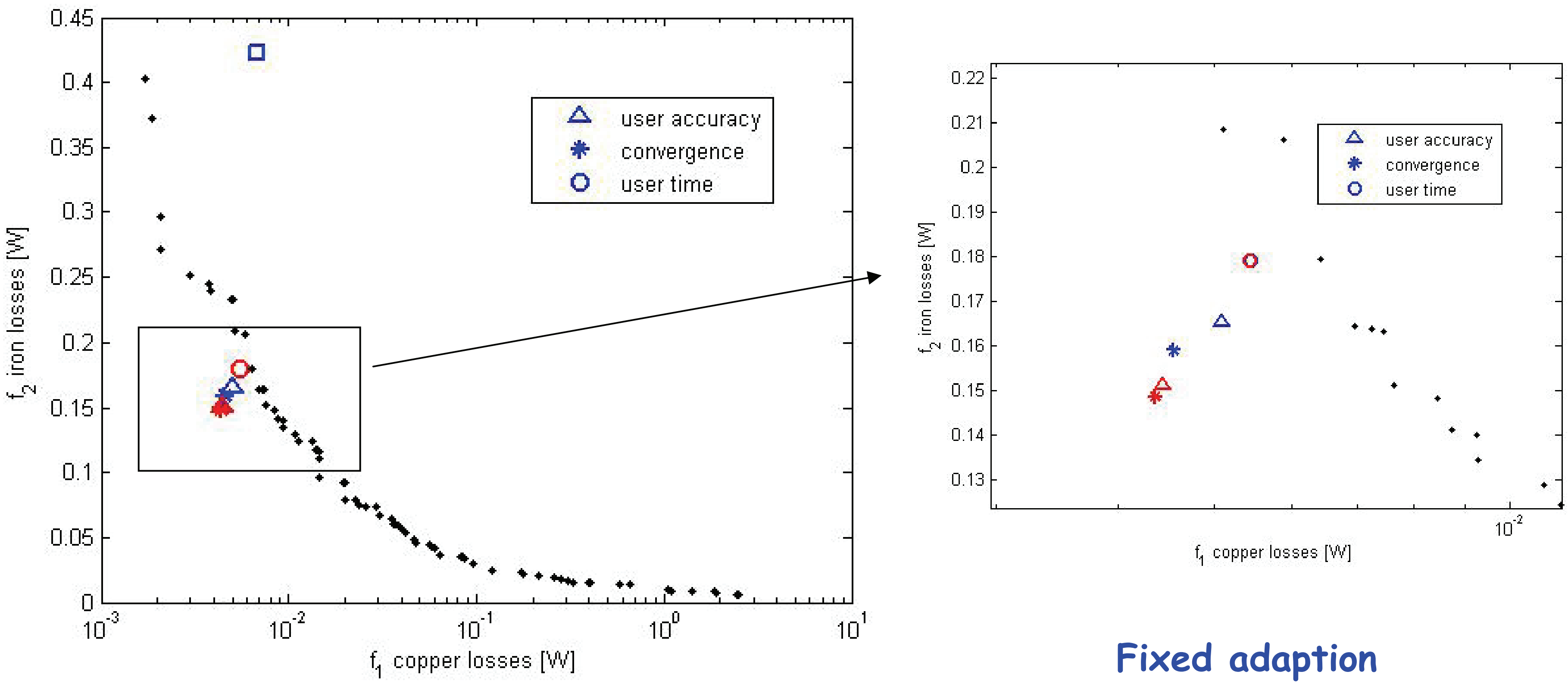
$x=[9.70, 1.21, 2.40, 7.93, 3.00]$ mm

$f_1=5.5$ mW, $f_2=179.3$ mW

11 iterations done in 1 hour

Improvement: 20% for f_1 , 60% for f_2

CASE STUDY



Fixed adaption

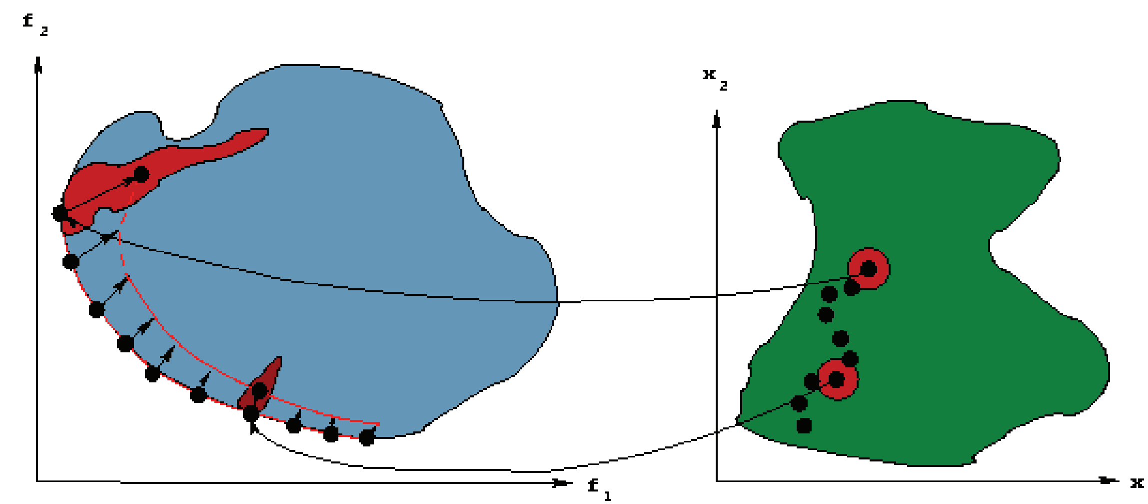
Variable adaption

ADVANCED EMO STUDIES: BENCHMARKING

Some goals of benchmarking :

- Move from test problems to industrial benchmarks
- Investigate topological properties of the Pareto-optimal front (convex/non-convex, connected/non-connected, uniformly/non-uniformly spaced)
- Define suitable metrics to measure the distance of a given solution point from the front
- Evaluate the sensitivity of the front to perturbation in the design space

Which front is robust ?

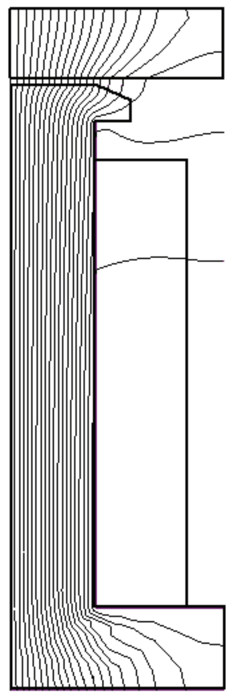


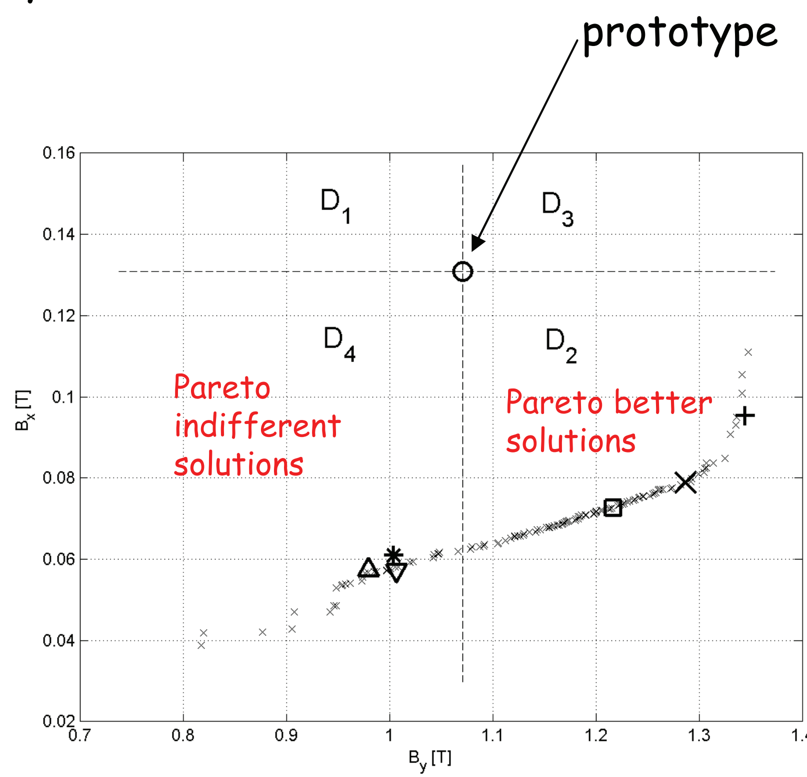
- Handle non-comparable solutions properly

Shape design of a magnetic pole

maximise B_y in the air gap

minimise B_x in the winding





In EMO, evaluating the performance of an optimization algorithm and assessing results is a challenging task.

Possibly, the multiobjective formulation of TEAM problems 22 and 25 could be improved.

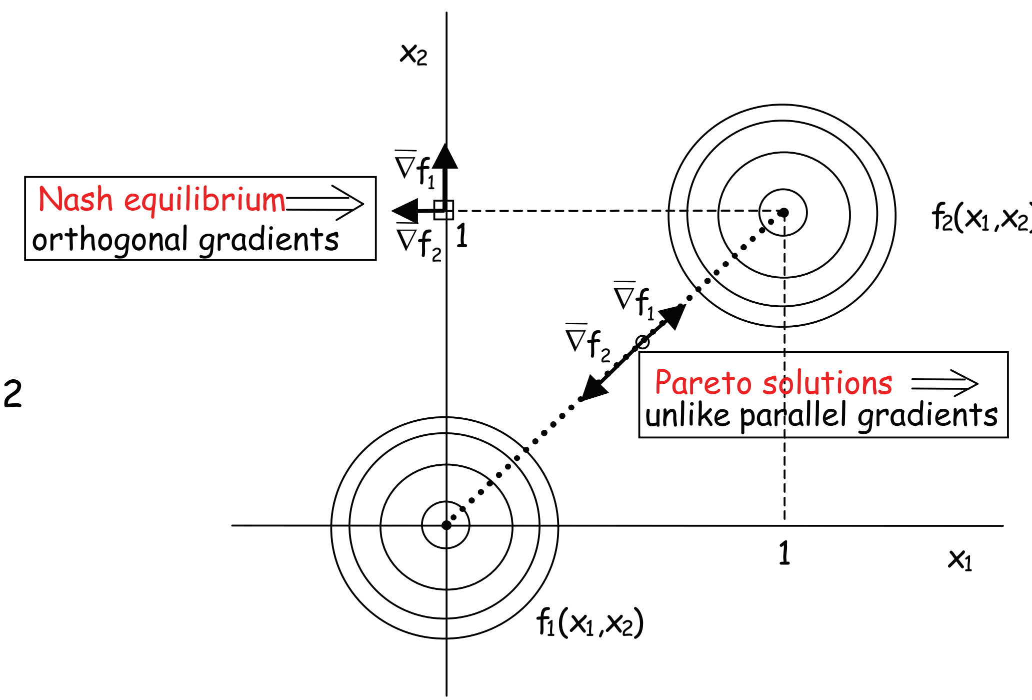
ADVANCED EMO STUDIES: NASH GAMES

An *a priori* method to provide the designer with a single optimum.

Each player minimises his own objective by varying a single variable and assuming that the values of the remaining p-1 objectives are fixed by the other p-1 players.

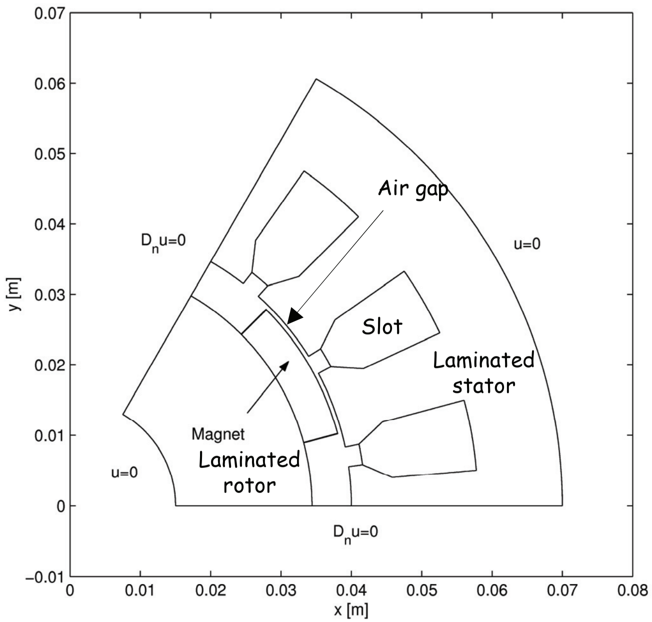
2D test case

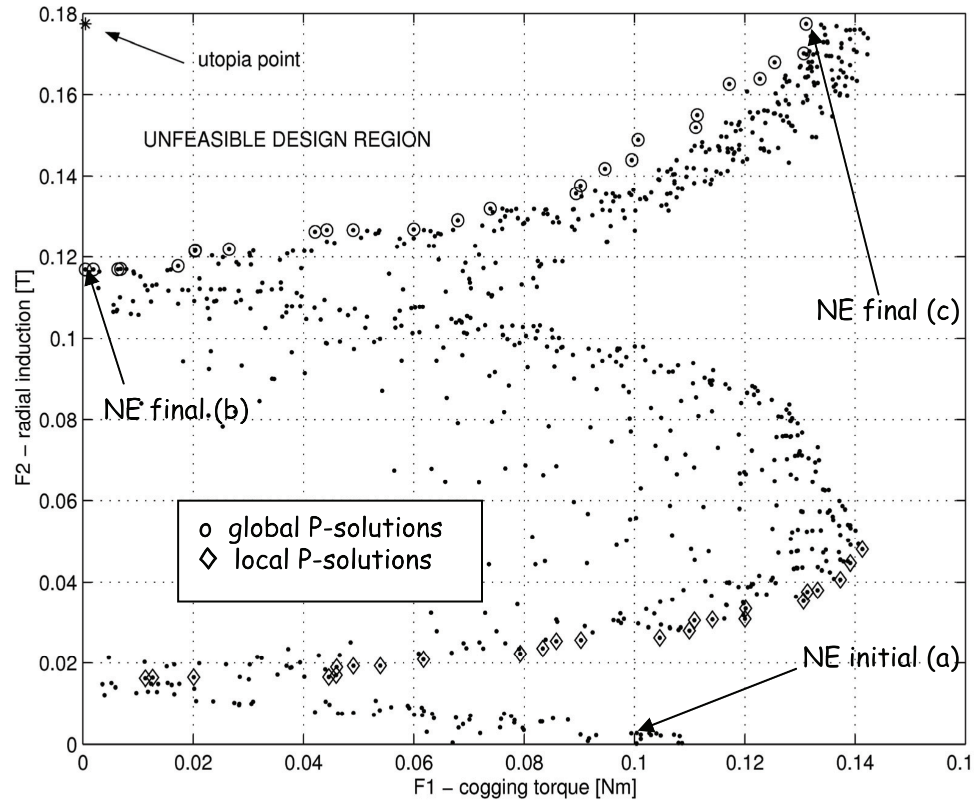
The game is over when neither player 1 nor player 2 can further improve their objectives.



NE is characterized by orthogonal gradients ⇨ uncorrelated disturbances ⇨ conditionally robust design.

PM three-phase motor





Design variables :
height and width of magnet

Objectives :
cogging torque (to be min), air-gap radial induction (to be max)

P. Di Barba, *Strategies of game theory for the automated optimal design in electromechanics*, Intl J of Applied Electromagnetics and Mechanics 27 (2008) 1-21

ADVANCED EMO STUDIES:
from static to dynamic Pareto fronts

In problems of shape design of electromagnetic devices, objectives and constraints usually do not depend on time

⇕

Static multiobjective optimisation

When either the objectives or the constraints, or both of them, depend on time, the PF is time dependent too

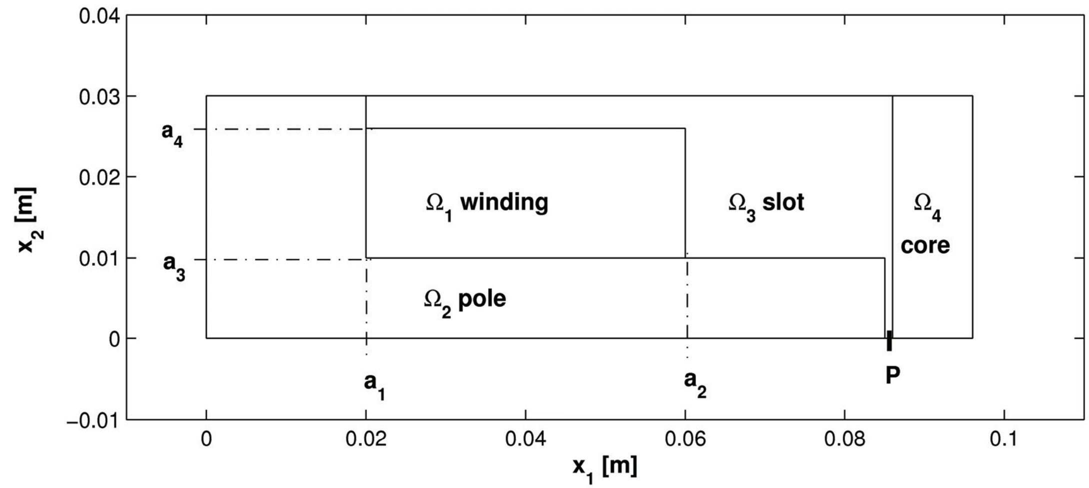
⇕

Dynamic multi-objective optimisation

Optimal control of the geometry of a magnetic pole, generating a prescribed induction field under step excitation of current

Design variables :

(a_1,a_2,a_3,a_4)



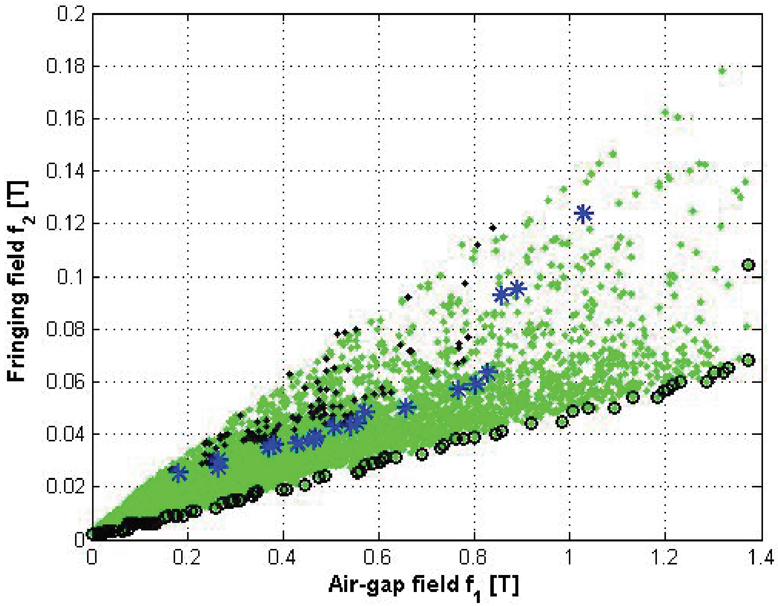
The problem reads: find the time-dependent family of non-dominated solutions from $t = 0^+$ to steady state such that

- air-gap induction is maximum
- power loss in the winding is minimum

under the constraint that

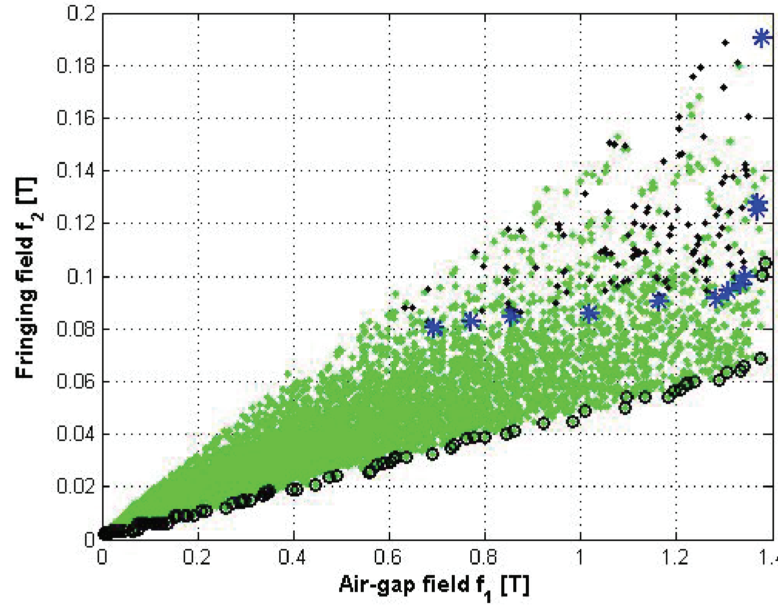
the power loss in the pole and the core at a given time instant ($t=10^{-2}\tau$) is not greater than the power loss in the winding.

Objective space at $t = \tau$



time-unconstrained (circle) and time-constrained (star) fronts

Objective space at steady state



time-unconstrained (circle) and time-constrained (star) fronts

The energy constraint, active at the beginning of the transient magnetic diffusion, affects the field performance at any subsequent time instant !

P. Di Barba, A. Lorenzi, A. Savini, *Dynamic Pareto fronts and optimal control of geometry in a problem of transient magnetic diffusion*, IET Science, Measurement and Technology, 2008, vol.2, no.3, 114-121

CONCLUSION

- In computational electromagnetism, the principle of natural evolution inspired a large family of algorithms that, through a procedure of self adaptation in an intelligent way, lead to an optimal result.

- Mathematics and biology, in a sense, exchange their reciprocal role in a sort of feedback operation: not just mathematics for biology, but also biology for mathematics; not only learning nature but also learning from nature.

- Perhaps the ultimate goal, which is still ambitious, is to get "intelligent" computing machines, capable of solving problems without explicitly programming them. This is the current challenge of artificial intelligence.

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In Search of Common Optimality Properties

Fritz-John Necessary Condition:

Solution x^* satisfy

1. $\sum_{m=1}^{M} \lambda_m \nabla f_m(x^*) - \sum_{j=1}^{J} u_j \nabla g_j(x^*) = 0,$ and

2. $u_j g_j(x^*) = 0$ for all $j = 1, 2, 3, \ldots\ldots, J$

- Requires differentiable objectives and constraints

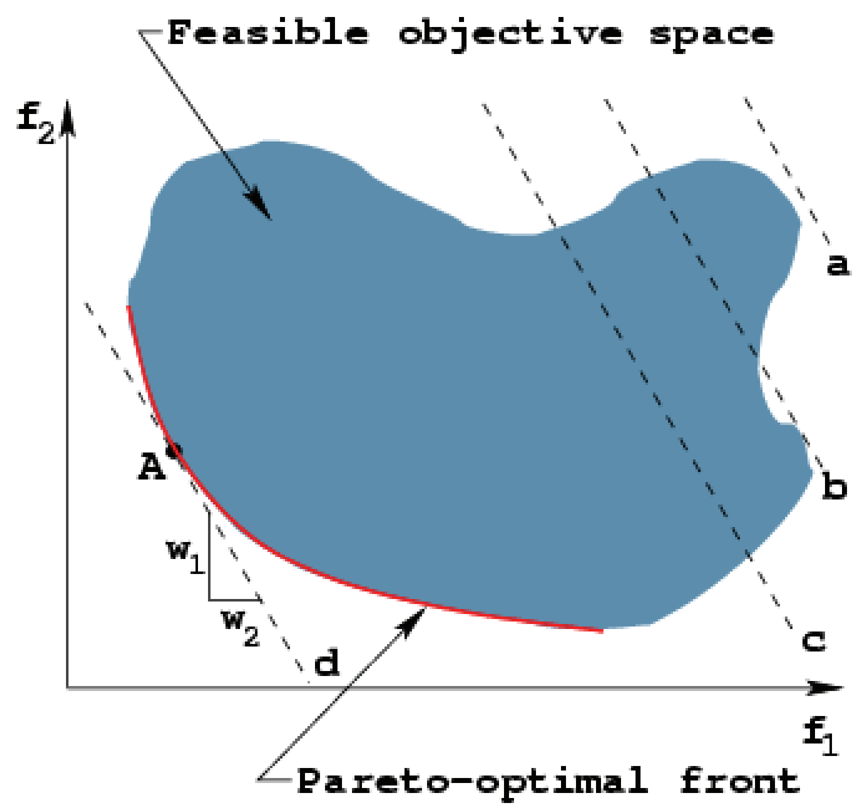
- It outlines the existence of some properties among Pareto-optimal solutions

Classical Approach: Weighted Sum Method

- Construct a weighted sum of objectives and optimize

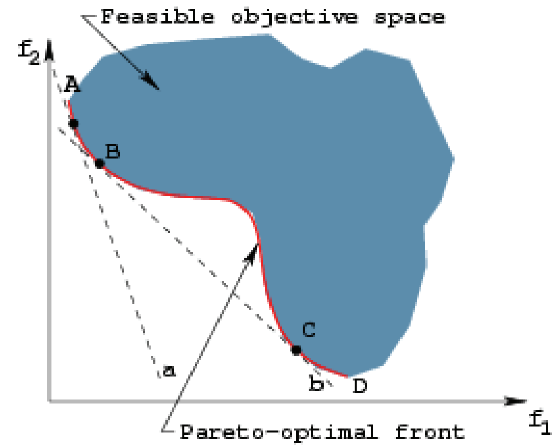
$$F(x) = \sum_{i=1}^{M} w_i f_i(x)$$

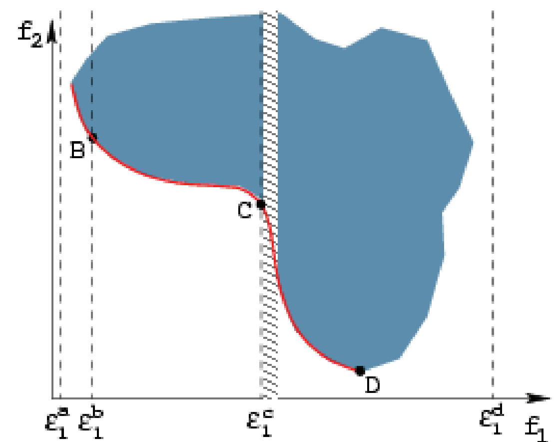
- User supplies weight vector *w*



Difficulties with Classical Methods

- Nonuniformity in Pareto-optimal solutions
- Inability to find some solutions
- Epsilon-constraint method still requires an ε-vector





Constraint handling

- Penalty function approach

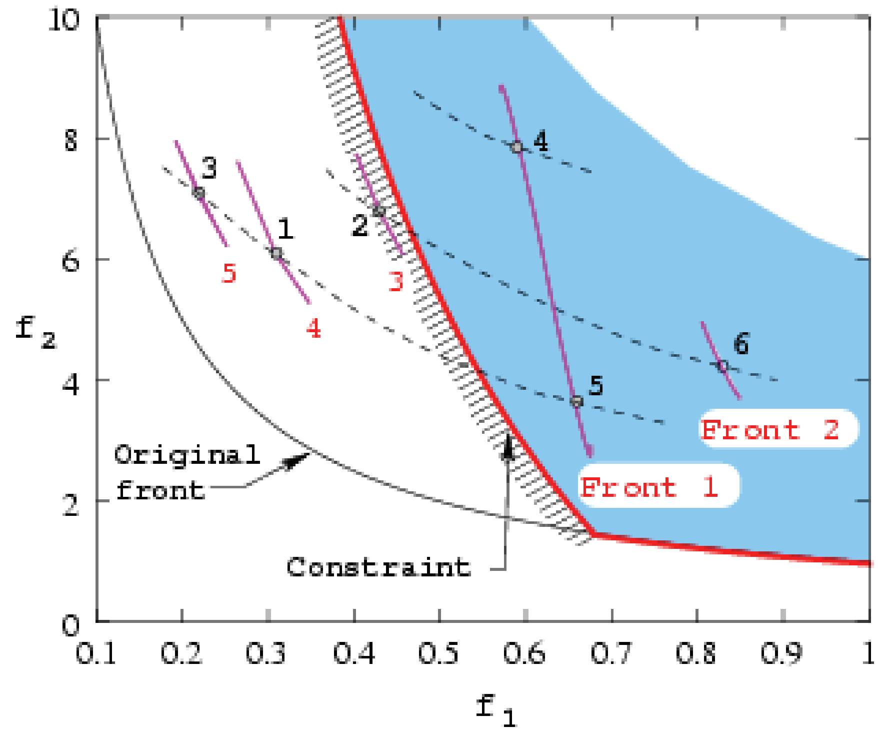
$$F_m = f_m + R_m \Omega\left(\vec{g}\right)$$

- Explicit procedures to handle infeasible solutions
- Modified definition of domination

Constraint-Domination Principle

A solution i **constraint-dominates** a solution j, if any is true:

1. i is feasible and j is not
2. i and j are both infeasible, but i has a smaller overall constraint violation
3. i and j are feasible and i dominates j



Identifying multiple Pareto-optimal solutions for choosing one implies a better process of decision making. In fact:

- Reveal common properties among optimal solutions
- Understand what causes trade-off
- Identify non-standard solutions to the design problem

↓

innovative design

- Fulfil *a posteriori* unpredicted constraints (it happens in real-life engineering)

↓

flexible design

Nash game: numerical implementation

Player 1 optimizes $f_1(x_1,x_2)$ acting on x_1 and receiving x_2 from player 2 at the previous iteration; then, player 1 sends the result to player 2.

Player 2 optimizes $f_2(x_1,x_2)$ acting on x_2 and receiving x_1 from player 2 at the previous iteration; then, player 2 sends the result to player 1.

The game is over (Nash equilibrium) when neither player 1 nor player 2 can further improve their objectives.