Non-invasive Thermometry for the Thermal Ablation of Liver Tumor: a Computational Methodology

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INTRODUCTION

Radiofrequency thermal ablation has became an important treatment of primary and metastatic liver tumor.

Radiofrequency generator 450 kHz

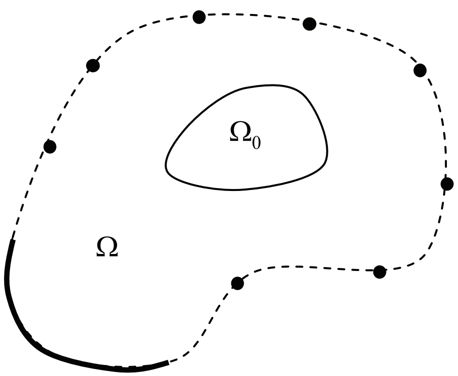
Needled electrode

Energy delivered to the tumor

Therapeutic temperature (50-100 °C)

During the thermal treatment, it is not possible to measure the temperature reached in the tissues by means of a non-invasive technique. From the clinical viewpoint, knowing the internal thermal field is of utmost interest in order to control power deposition for having an effective treatment.

PROBLEM STATEMENT (1/2)



— Γ_0 (continuous)

— Γ_1 (continuous, temperature known)

• Γ_2 (discrete, temperature known)

- - - Γ_3 (continuous, convection exchange)

$\Gamma_1 \cap \Gamma_2 = 0$
$\Gamma_1 \cap \Gamma_3 = 0$
$\Gamma_2 \cap \Gamma_3 = \Gamma_2$
$\Omega_0 \subset \Omega$

Γ_1: electrically grounded electrode, Ω_0 : target volume (i.e. the tumoral mass)
Γ_2: position of the temperature probes

Direct thermal problem: $-\nabla \cdot (k \nabla u) = f$ in Ω, with $f = \sigma E^2 - c_b w_b (u - u_b)$

Linear, well-posed problem.
Solution: estimated temperature field all over the domain Ω.

Boundary conditions:

$u = U$ along Γ_1

$-k \dfrac{\partial u}{\partial n} = h(u - u_0)$ along Γ_3

u: temperature (°C), k: thermal conductivity ($Wm^{-1}°C^{-1}$), f: source term (Wm^{-3}),
σ: electric conductivity (Sm^{-1}), E: impressed electric field (Vm^{-1}),
c_b: specific heat of the blood ($J\,kg^{-1}°C^{-1}$), w_b: mass flow rate ($kg\,s^{-1}$),
u_b: temperature of the blood (°C), h: convection coefficient n: normal unit vector.

PROBLEM STATEMENT (2/2)

Inverse problem:

Knowing the geometry of the domain Ω, the tissue properties (k, σ, c_b, w_b, h), the source term f, the boundary conditions along Γ_1 and Γ_3 and the supplementary condition along Γ_2, find the temperature field u in the domain $\Omega \backslash \Omega_0$ and along the boundary Γ_0.

Solution: reconstructed temperature field over the domain $\Omega \backslash \Omega_0$

The uncertainty of tissue data and the steady-state typology of the direct problem make the continuation problem **ill-posed**, because it is impossible to guarantee the uniqueness of a solution a priori.

SOLUTION STRATEGY

Minimization strategy

Defining the error functional $F(\lambda) = \left\| u(\lambda)|_{\Gamma_2} - u^*|_{\Gamma_2} \right\|_2$, starting from a guess solution,

find λ^* such that $F(\lambda^*) = \inf_{\lambda} F(\lambda)$ where $u(\lambda)|_{\Gamma_2}$ is the reconstructed field along Γ_2

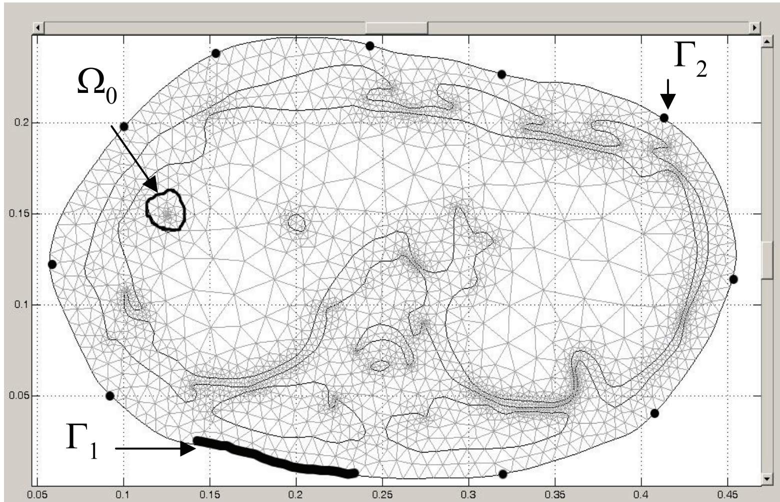
$u^*|_{\Gamma_2}$ is the supplementary condition.

$\lambda \in \Re^1 \Longleftrightarrow \Gamma_0$ supposed to be isothermal

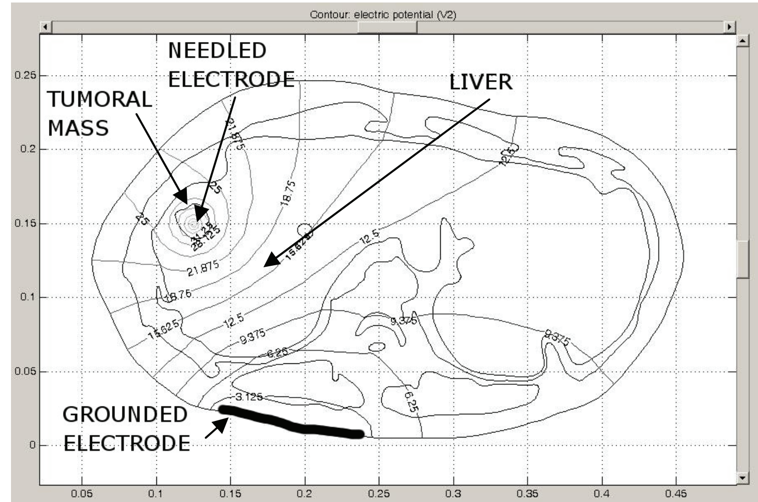
Given a value of λ, the direct problem is **well-posed**: a finite-element analysis of the following problem is performed and the error functional $F(\lambda)$ is updated.

$$
\begin{array}{ll}
-\nabla \cdot (k \nabla u) = f & \text{in } \Omega \backslash \Omega_0 \\
u = U & \text{along } \Gamma_1 \\
-k \dfrac{\partial u}{\partial n} = h(u - u_0) & \text{along } \Gamma_3 \\
u = \lambda & \text{along } \Gamma_0
\end{array}
$$

CASE STUDY



Finite-element model
Mesh: 2,990 nodes and 5,848 first order-triangular elements



Electric potential lines [V]

Electromagnetic problem

The electric potential of the needled electrode is set to 50 V with respect to the grounded electrode Γ_1

Γ_3 : condition of tangential electric field

Source term of the thermal problem

Basic problem:
convection coefficient
$h = 12\ W\ m^{-2}\ °C^{-1}$
$U = 37°C$

Perfusion coefficient w ($kg\ s^{-1}\ m^{-3}$)	Liver	Fat	Muscle	Tumor
	0.416	0.36	0.45	16.67

NUMERICAL RESULTS

ℓ^2 error over a domain D: $\varepsilon(D) = [(u_r - u_e)^T (u_r - u_e)]^{1/2}$

$u_r(i)$ $i = 1,..,n_p$: reconstructed temperature field, $u_e(i)$: estimated temperature field
n_p: number of nodes of the grid discretizing D.
$\varepsilon_1 \equiv \varepsilon(\Omega)$ $\varepsilon_2 \equiv \varepsilon(\Omega \backslash \Omega_0)$

Basic problem

Estimated temperatures (°C)	Reconstructed temperature (°C)
40.191	40.895
42.568	
44.882	
42.207	
42.996	
40.126	
41.069	
42.915	
42.621	
42.655	

Numerical results varying some thermophysical properties

Variation	ε_1 (°C)	ε_2 (°C)	Reconstructed temperature of Γ_0 (°C)
Basic problem	6.1210	1.1637	40.8951
h = 20 ($W\ m^{-2}\ °C^{-1}$)	6.1324	1.1659	40.8898
h = 30 ($W\ m^{-2}\ °C^{-1}$)	6.1415	1.1677	40.8853
w’ = 0.5 w ($kg\ s^{-1}\ m^{-3}$)	9.1634	2.5031	43.8645
w’’ = 2 w ($kg\ s^{-1}\ m^{-3}$)	3.6620	0.4458	39.0483

Depedence of numerical results on the starting point of the minimization

Starting temperature of Γ_0 (°C)	ε_1 (°C)	ε_2 (°C)	Reconstructed temperature of Γ_0 (°C)	Final residual of the error functional (°C)
10	6.1210	1.1637	40.8951	$1.5827\ 10^{-4}$
30	6.1064	1.1611	40.8952	$1.7150\ 10^{-4}$
70	6.1066	1.1611	40.8952	$1.7118\ 10^{-4}$
95	6.1197	1.1635	40.8951	$1.5831\ 10^{-4}$

CONCLUSION

- The problem of determining the temperature in a subdomain and along its boundary has been formulated as an optimisation problem, minimizing a suitable functional.

- Convergence of the numerical solution has been obtained, starting from various guess solutions for the temperature along the boundary of the subdomain.

- The methodology can be usefully applied for the prediction of the temperature of tumoral tissues during thermal ablation treatment.