Simulation-Based Design in Electrical Engineering

1. Simulation-Based Design in Engineering Praxis
2. Dielectric Design of HV Products
3. **Magnetic Problems in Engineering Praxis**
4. Coupled Problems
5. Optimization 1
6. Optimization 2

Magnetisms: probably one of the most important physical phenomena for the existence of the human being

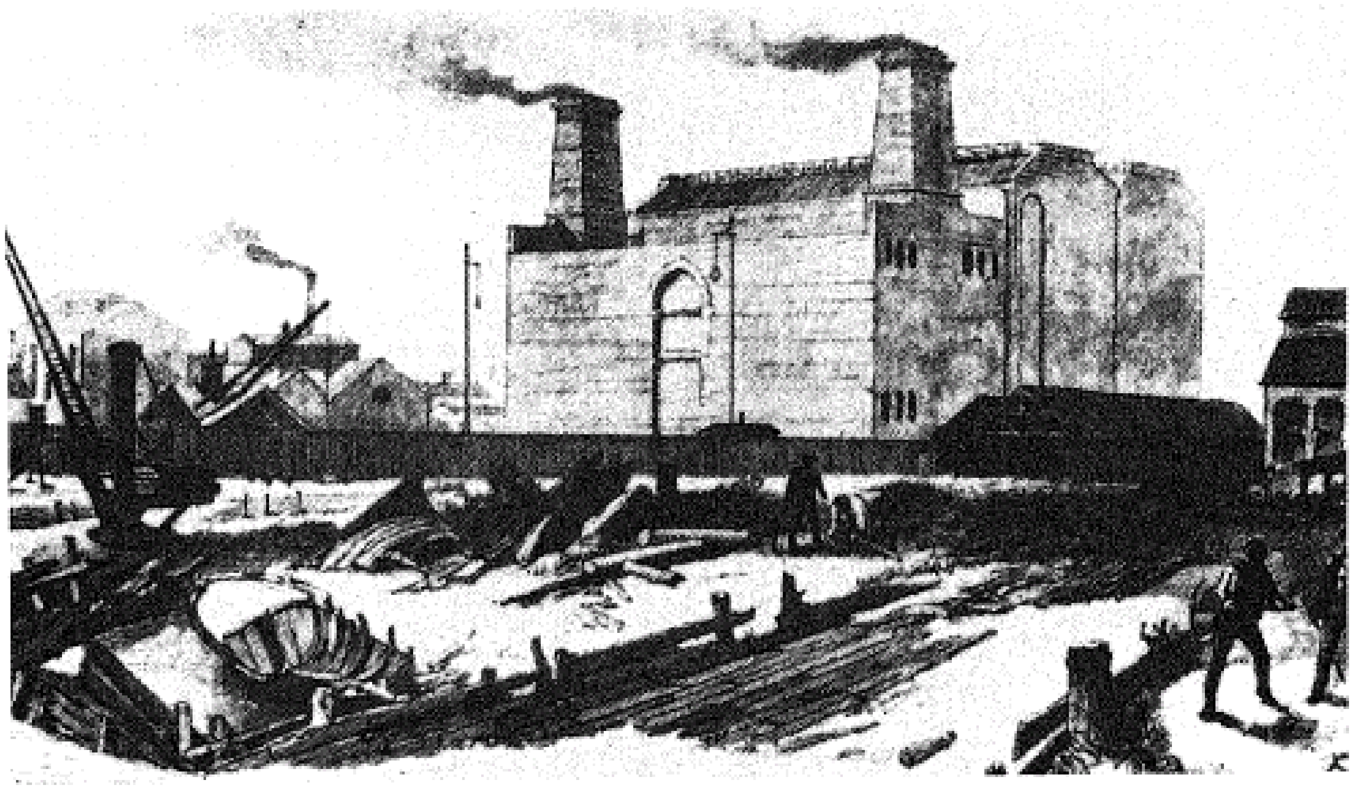


Figure 1. Deptford in construction around 1890. From Notes and Records of the Royal Society of London, 1964, 19:37.

- *1890: Sebastian de Ferranti* launch 10.000V Deptford system to supply 38.000 lamps in London
- Shortly after, a mysterious fire destroyed all the transformers at a substation near London putting it out of commission for the entire winter

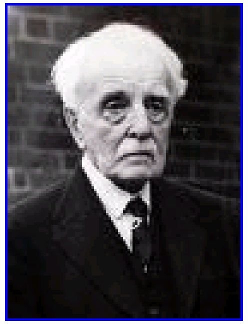
1995 by The History of Science Society, 86:30-51

John Ambrose Fleming

Sir John Ambrose Fleming

(Resonance theory)

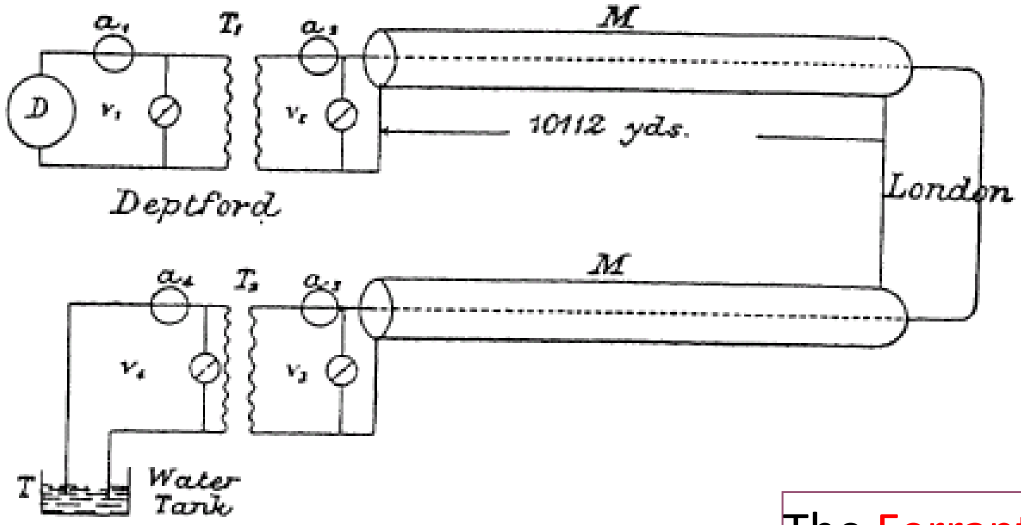
James Swinburne



Sir **James Swinburne**

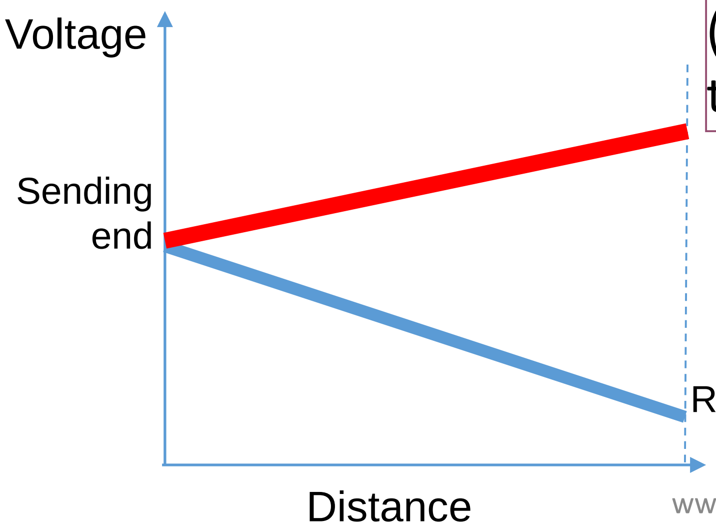
(Armature reaction)

Ferranti effect

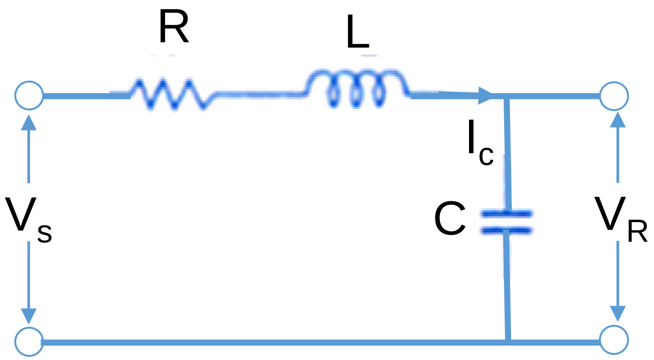


Fleming's controlled field experiment on the Ferranti effect *(Journal of the Institution of Electrical Engineers, 1891, 20:389)*

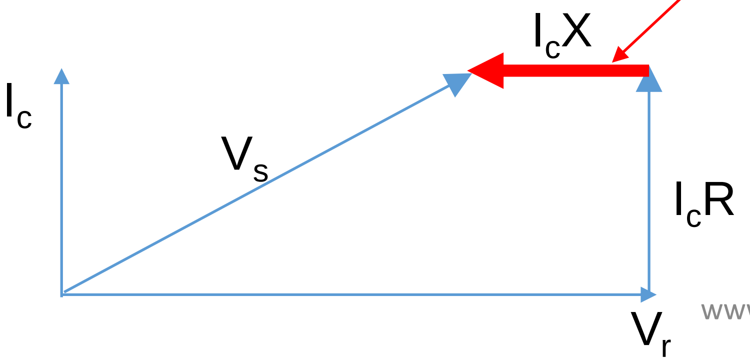
The Ferranti Effect is a rise in voltage occurring at the **receiving end** of a (long) transmission line, comparing to the voltage at the **sending end**



Ferranti effect



The charging current I_c produces drop in the reactance of the line which is in phase opposition to the receiving end voltage and hence **the sending end voltage Vs becomes smaller than the receiving end voltage Vr**

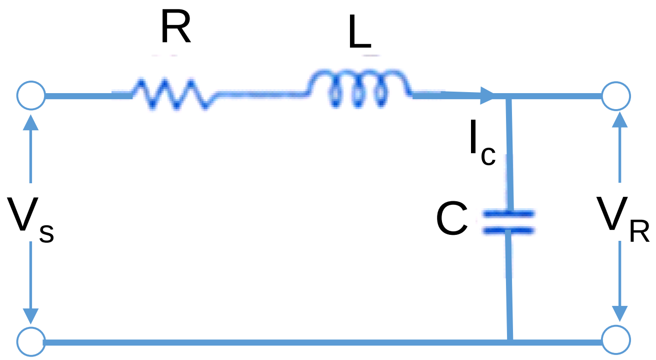


$$I_c = \omega C U \; 10^{-6}$$

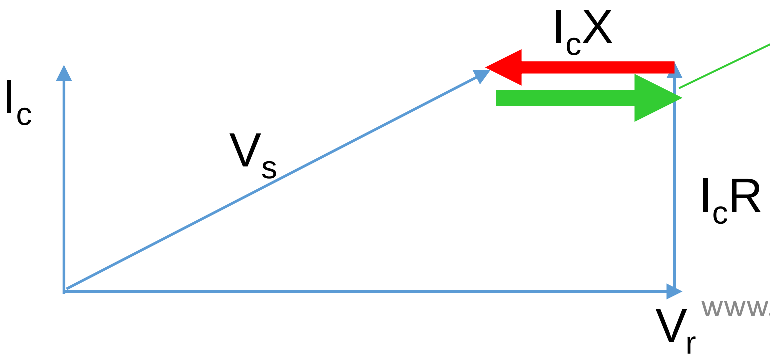
where:

- Ic = charging current (A/km)
- ω = 2πf ; f = System frequency
- C = capacitance per unit length (μF/km)
- U = Applied voltage (V)

Ferranti effect



Solution for Ferranti effects are the **shunt reactors** added at the end of the lines to compensate the capacitive reactive energy



$$I_c = \omega C U \; 10^{-6}$$

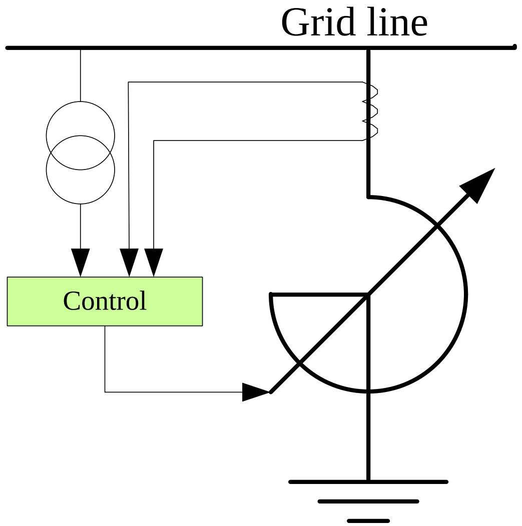
where:

- I_c = charging current (A/km)
- $\omega = 2\pi f$; f = System frequency
- C = capacitance per unit length (µF/km)
- U = Applied voltage (V)

What is controllable reactor?

Shunt reactor (150Mvar, 400kV)

Shunt reactor (700Mvar, 154kV)



- Basic function
  - Inductance control
  - Reactive power control
  - Advanced System control
- Requirements
  - Fast Speed
  - Large control range
  - Low harmonics

Magnetic Design

Static problems

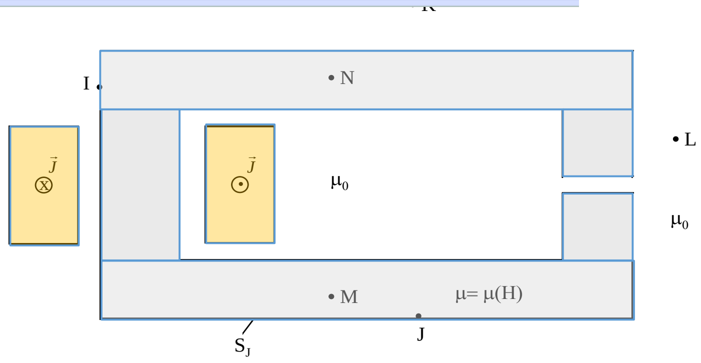
Formulation:

$$rot\,\vec{H} = \vec{J} \quad div\vec{B} = 0$$

$$H_t^i = H_t^0, \quad B_n^i = B_n^0$$

$$\vec{B} = \mu\vec{H}$$ *(isotropic, linear)*

$$\vec{B} = \mu(H)\cdot\vec{H}$$ *(isotropic, nonlinear)*



Equivalent surface charges

Equivalent volume charges

II Fredholm Integral Equation

$$2\pi\frac{\sigma(I)}{\lambda(I)} - \int_S \sigma(J)\frac{(\boldsymbol{r}_{IJ},\boldsymbol{n}_I)}{r_{IJ}}dS_J = 4\pi\cdot H_n^{ext}(I) + \int_{V_N}\rho(N)\frac{(\boldsymbol{r}_{IN},\boldsymbol{n}_I)}{r_{IJ}}dV_N$$

$$\left[\frac{\sigma}{\lambda}\right] - [A]\cdot[\sigma] = [B] + [D]\cdot[\rho]$$

Magnetic Design

Static problems

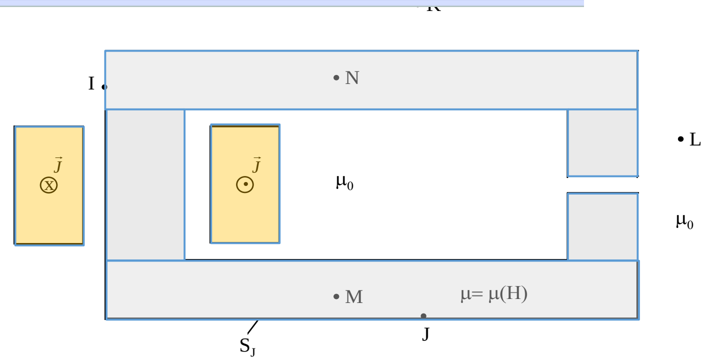
Formulation:

$$rot\,\vec{H} = \vec{J} \qquad div\vec{B} = 0$$

$$H_t^i = H_t^0, \quad B_n^i = B_n^0$$

$$\vec{B} = \mu\vec{H}$$ *(isotropic, linear)*

$$\vec{B} = \mu(H)\cdot\vec{H}$$ *(isotropic, nonlinear)*



The size of the matrix system is determined by the number of the surface unknowns!

Volume charge appears as a correction of the RHS during iteration process

$$\left[\frac{\sigma}{\boxed{\lambda}}\right] - \boxed{[A]}\cdot[\sigma] = [B] + [D]\cdot\boxed{[\rho]}$$

Material parameters appear only as a diagonal term!
Matrix generated only once!

$$\ldots \frac{\vec{r}_{MJ}}{r_{MJ}^3}\,dS_J + \frac{1}{4\pi}\int_{V_N}\rho(N)\frac{\vec{r}_{MN}}{r_{MN}^3}\,dV_N$$

$$L = \frac{1}{I_R}\int_{S_c}\mathbf{B}d\mathbf{s} \qquad e = -N\frac{d\Phi}{dt}$$

Magnetic Design

Static problems

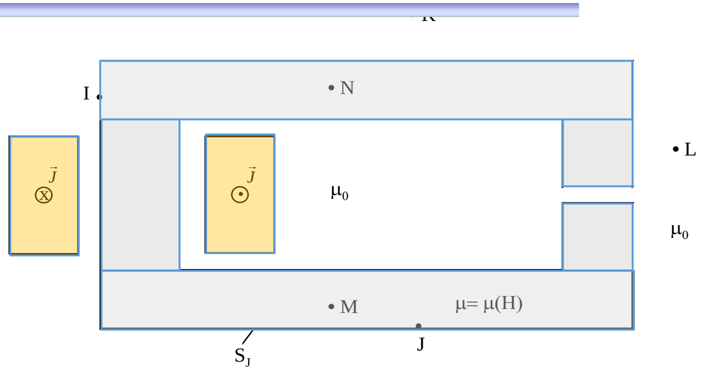
Formulation:

$$rot\,\vec{H} = \vec{J} \quad div\vec{B} = 0$$

$$H_t^i = H_t^0, \quad B_n^i = B_n^0$$

$$\vec{B} = \mu\vec{H}$$ *(isotropic, linear)*

$$\vec{B} = \mu(H)\cdot\vec{H}$$ *(isotropic, nonlinear)*

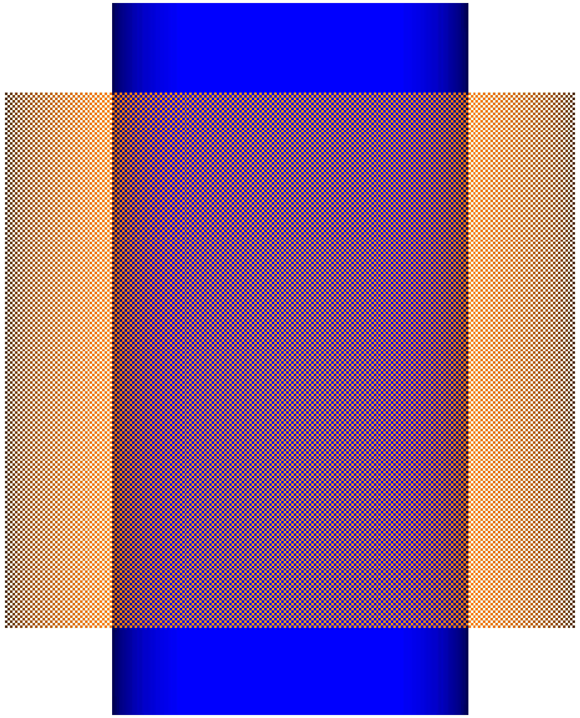


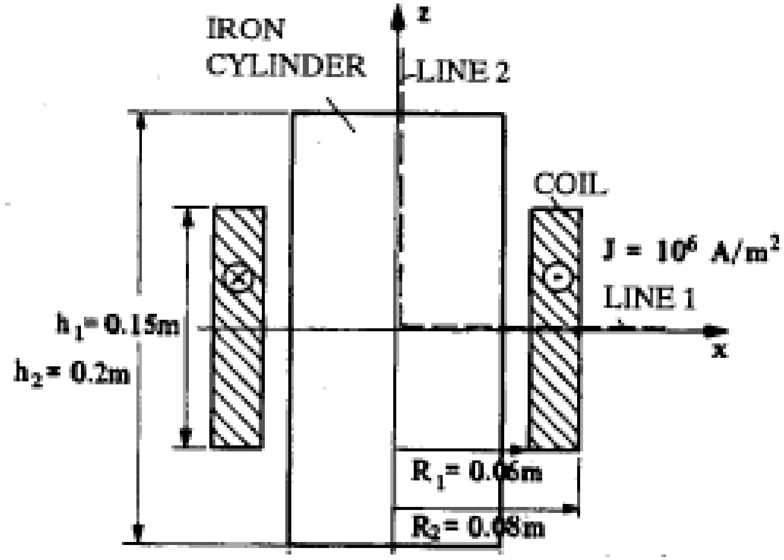
$$\vec{H}_{IRON} = \frac{1}{4\pi}\int_{S_J}[\sigma(J)-\sigma_\infty(J)]\frac{\vec{r}_{MJ}}{r_{MJ}^3}dS_J + \frac{1}{4\pi}\int_{V_N}\rho(N)\frac{\vec{r}_{MN}}{r_{MN}^3}dV_N$$

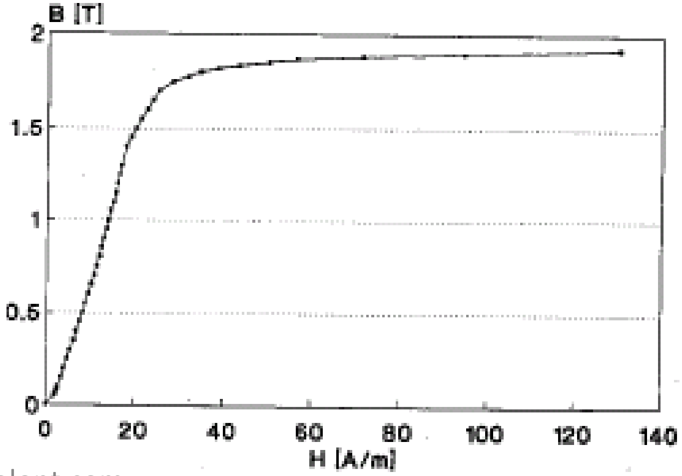
$$\vec{H}_{AIR} = \frac{1}{4\pi}\int_{S_J}\sigma(J)\frac{\vec{r}_{KJ}}{r_{KJ}^3}dS_J + \frac{1}{4\pi}\int_{V_N}\rho(N)\frac{\vec{r}_{KN}}{r_{KN}^3}dV_N + \vec{H}^J(K)$$

B. Krstajic, Z. Andjelic, S. Milojkovic, S. Salon: **Non-linear 3D Magnetostatic Field Calculation by the IEM with Surface and Volume Magnetic Charges,** Tran. on Mag., vol.28,No.2,March 1992

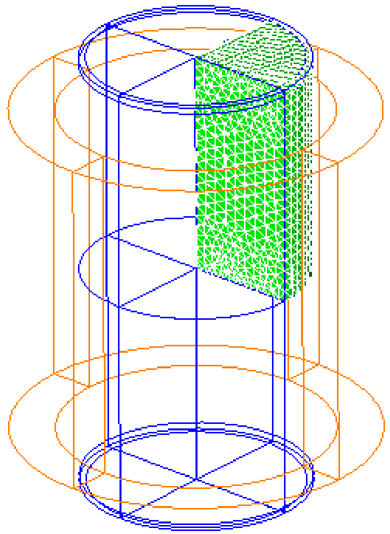
Test example: Team Benchmark problem, JIEE 1981

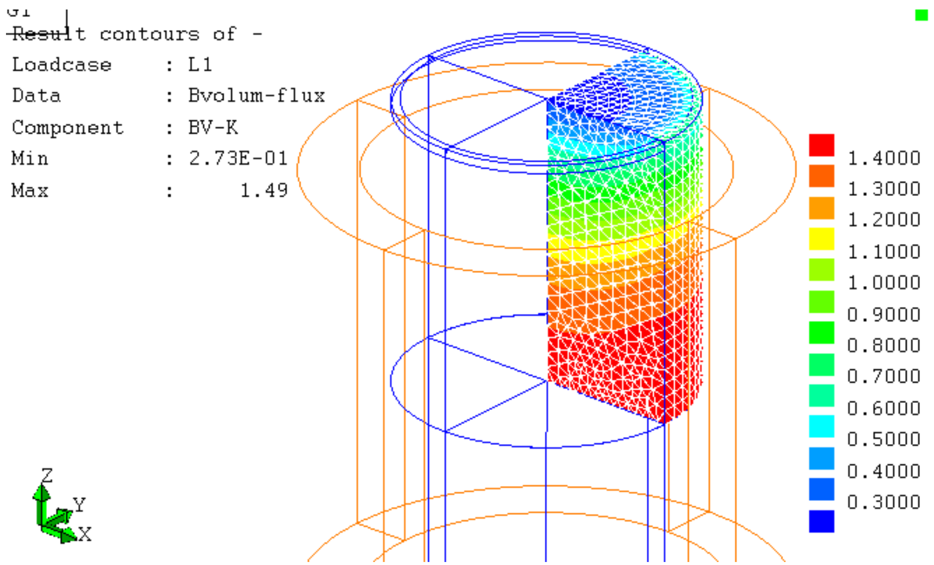


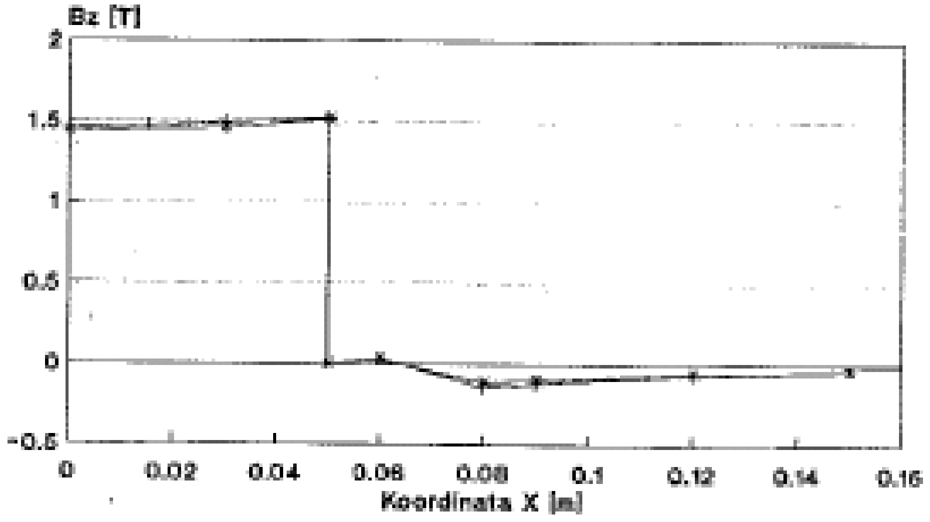




Test example: Team Benchmark problem, JIEE 1981

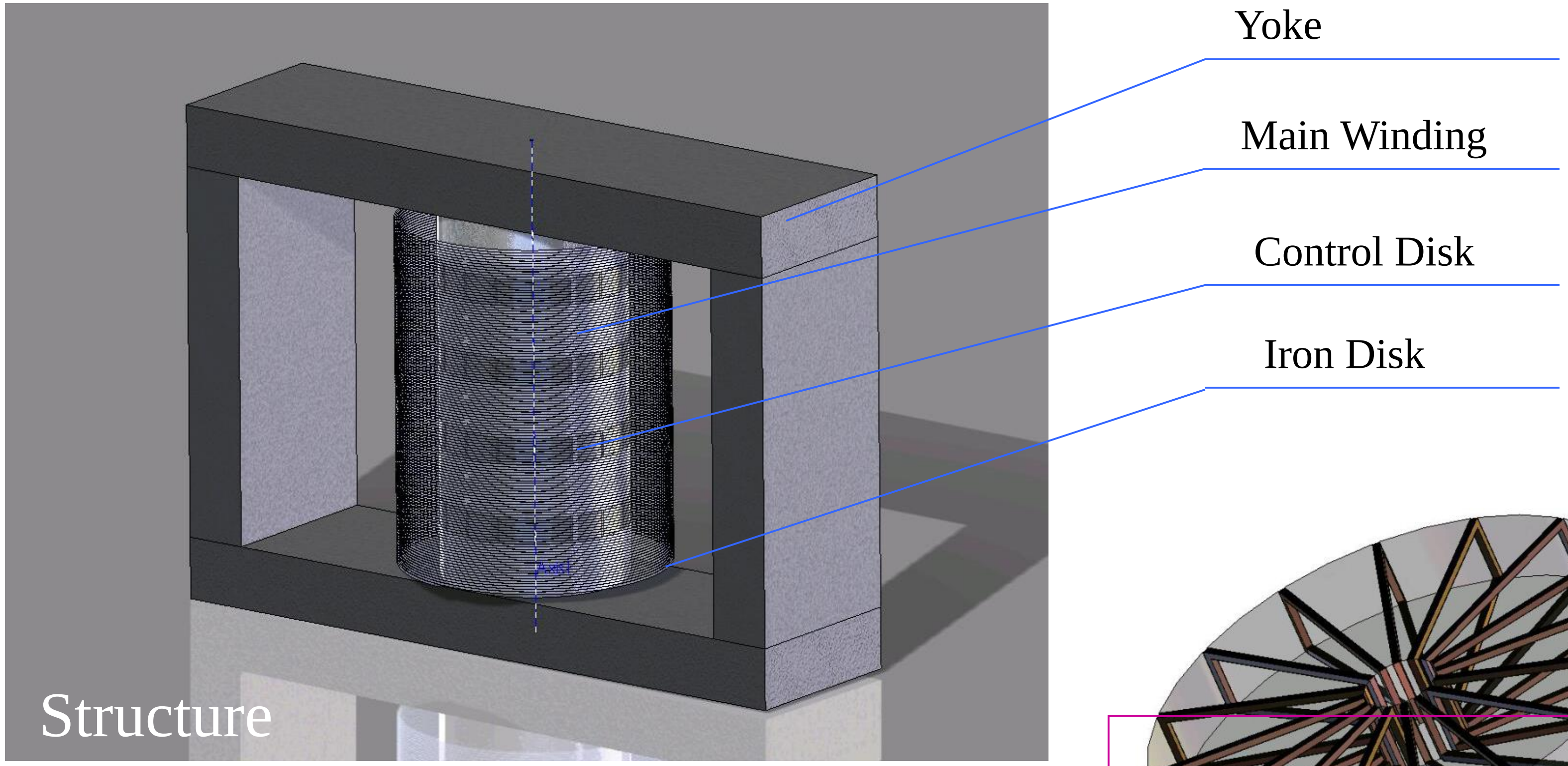


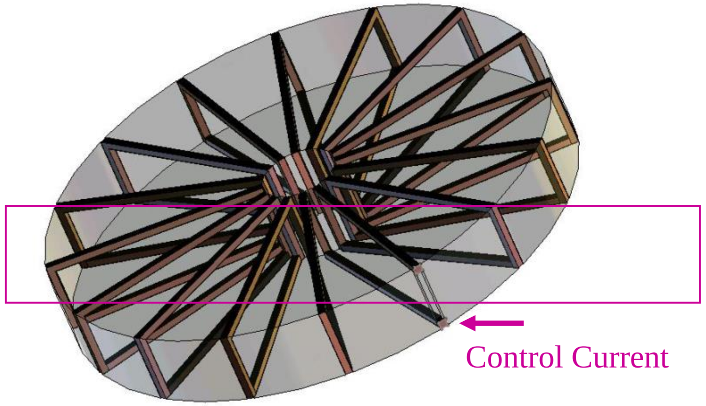




IEEE Tran. on Mag., vol.28,No.2,March 1992

What is controllable reactor? (cont.)

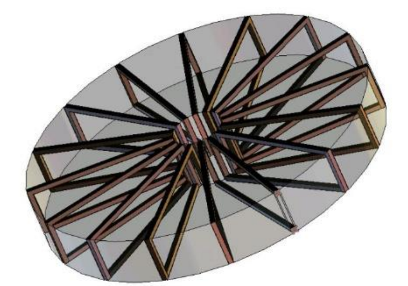


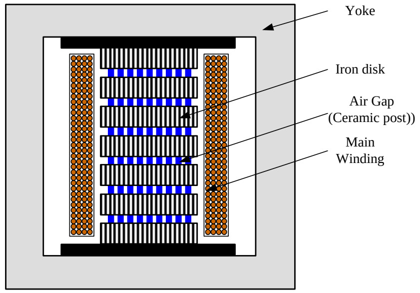


Control Disk

- Similar to a fixed reactor
- The control disks replace the air-gaps in the fixed reactor
- The control disks are with control winding in which DC currents flow through

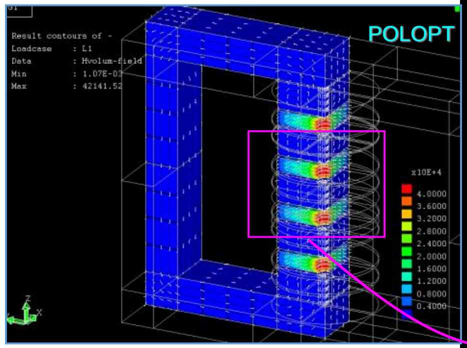
Controllable Reactor

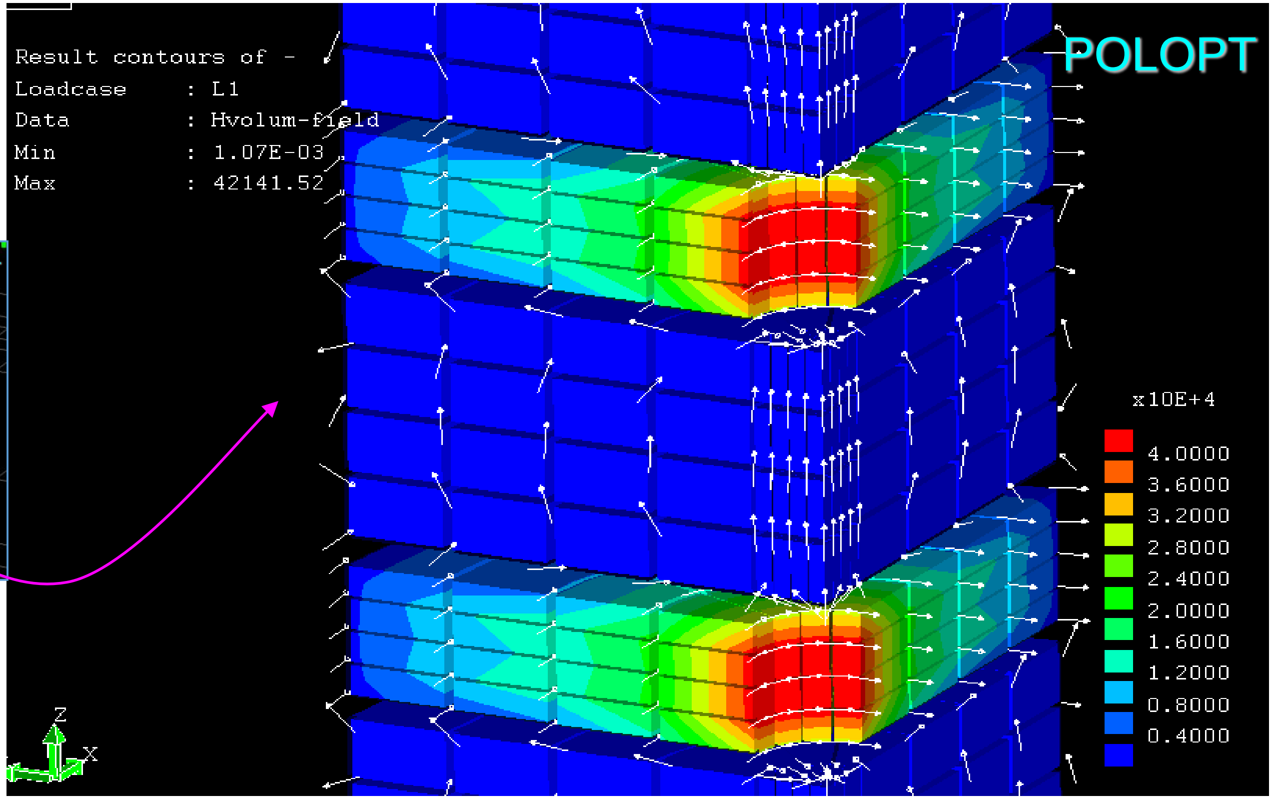




Model 1

Controlled Bias DC current $\uparrow \Rightarrow$ Saturation $\uparrow \Rightarrow \mu_r \downarrow \Rightarrow L \downarrow$



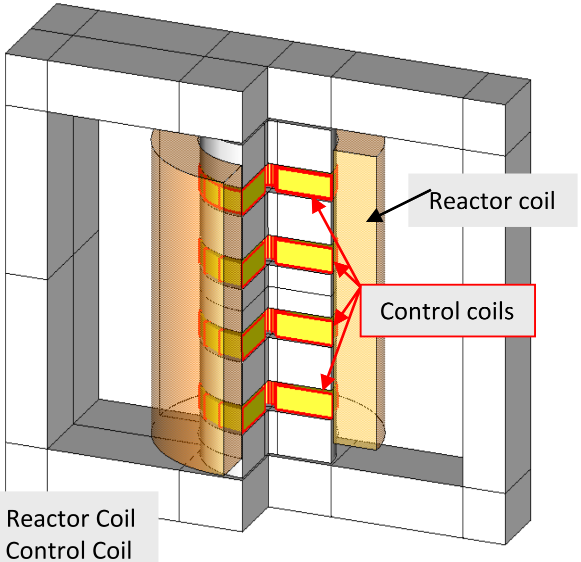


I_R = 1kA
I_C = 1kA

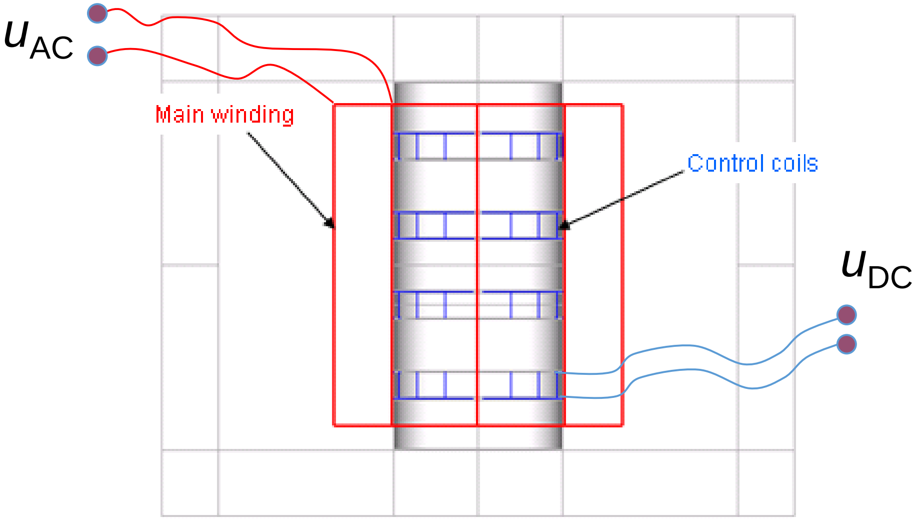
Magnetic field distribution

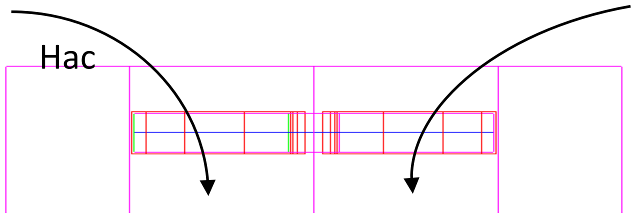
Controllable Reactor

Induced voltage calculation



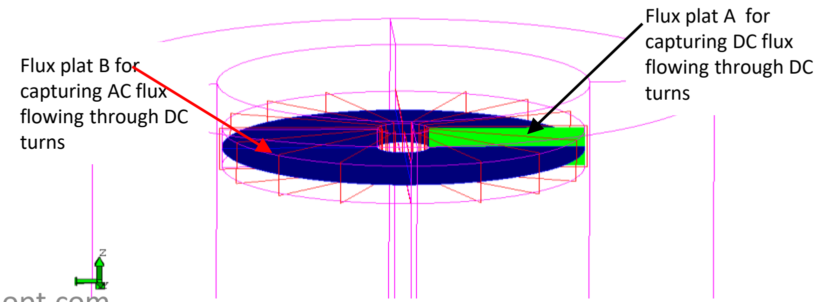
I_{RC} = Current in Reactor Coil
I_{CC} = Current in Control Coil



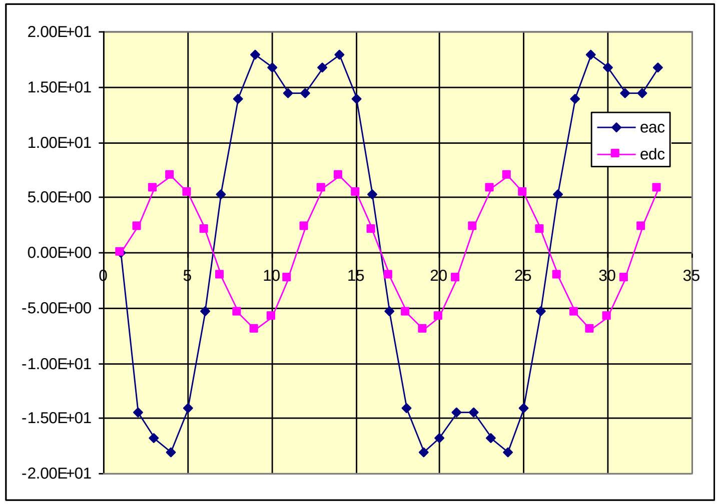


$$u_{AC} = -e_{AC} = i_{AC} \cdot R_{AC} + N_{AC} \frac{d\Phi_{AC}}{dt} + \cancel{N_{AC} \frac{d\Phi_{DC \to AC}}{dt}}$$

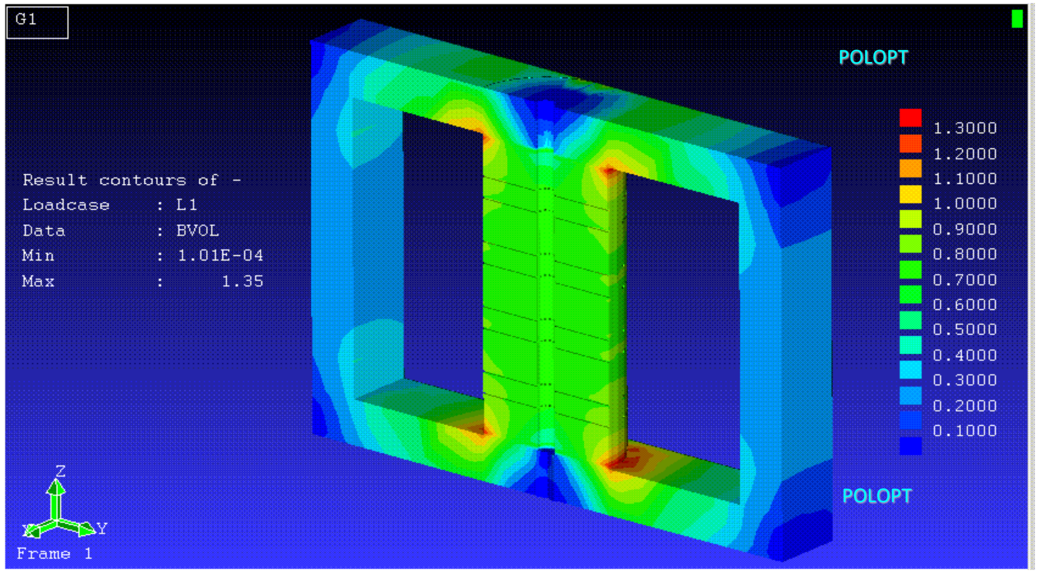
$$u_{DC} = -e_{DC} = i_{DC} \cdot R_{DC} + N_{DC} \frac{d\Phi_{DC}}{dt} + N_{DC} \frac{d\Phi_{AC \to DC}}{dt}$$



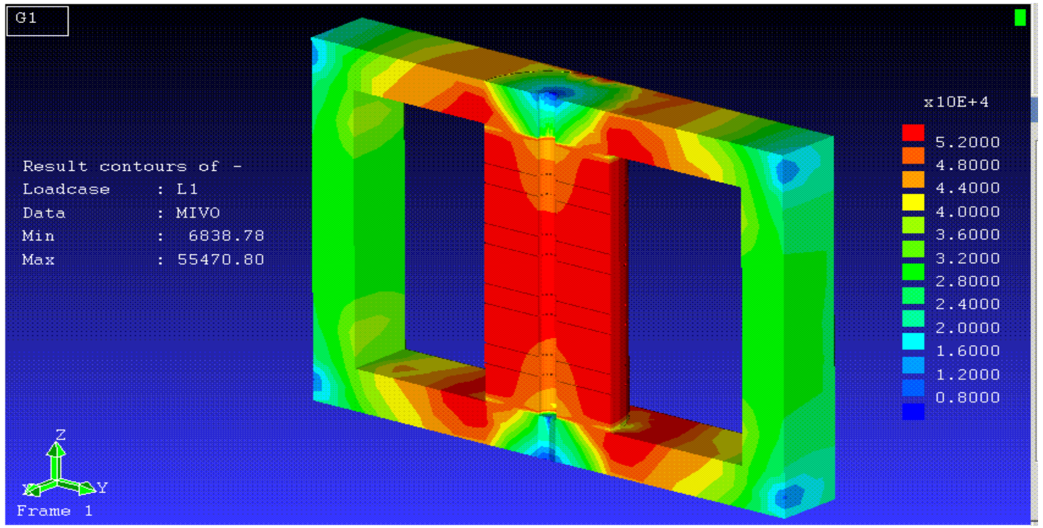
Example: Controllable Reactor



Induced voltages Uac, Udc



Flux change for I_{DC} = 0-1000A



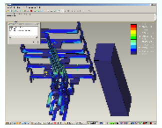
Permeability changes for I_{DC} = 0-1000A

Force Analysis

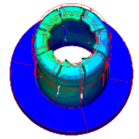
FORCE ANALYSIS

- BEM offers the most natural / accurate / efficient modeling approach for:

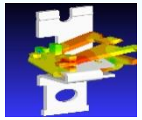
- Computation of the DC forces in current-currying bus-bars in the presence of:
  - linear magnetic materials
  - non-linear magnetic materials



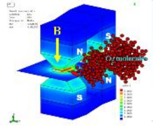
- Computation of the AC forces in current-currying bus-bars



- Computation of the forces acting on the magnetic bodies:
  - linear magnetic materials
  - non-linear magnetic materials

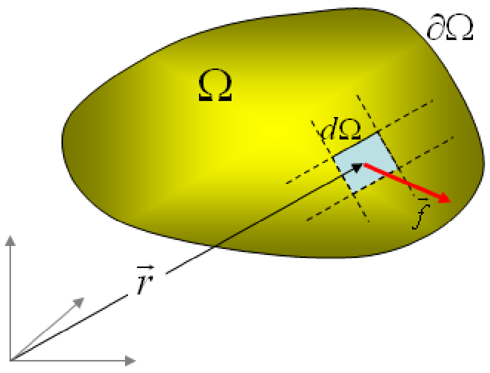


- Computation of the forces acting on the non-magnetic bodies



Force analysis

$$\vec{F} = \int_{\Omega} \vec{f}(r) d\Omega$$



J.R. Melcher: Continuum Electromechanics, M.I.T Press, Cambridge, MA 1981

M. Zahn: Derivation of the Korteweg-Helmholtz Electric and Magnetic Forces Densities Including Electrostriction and Magnetostriction from the Quasistatic Pointing Theorems, IEEE Conf. on El. And Diel. Phenomena, 2006, ISEN 1-4244-0546-7

Korteweg-Helmholtz force density in compressible magnetizable media:

$$\vec{f} = \mu(\rho)\vec{J}_f \times \vec{H} - \frac{1}{2}\vec{H}\cdot\vec{H}\nabla\mu(\rho) + \nabla(\frac{1}{2}\rho\frac{d\mu(\rho)}{d\rho}\vec{H}\cdot\vec{H})$$

Incompressible linear magnetizable media

$$\vec{f} = \mu_0\vec{J}_f \times \vec{H} + \mu_0\vec{M}\cdot\nabla\vec{H}$$

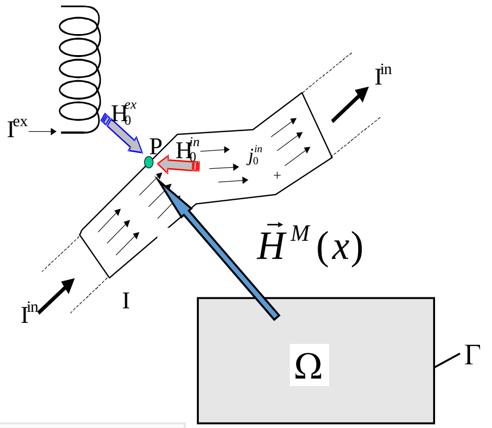
Incompressible, non-permeable media

$$\vec{f} = \mu_0\vec{J}_f \times \vec{H}$$

DC Forces on current-carrying conductors

$$\vec{F} = \int_{\Omega} \vec{f}(r)\, d\Omega$$

$$\vec{f} = \mu_0 \vec{J}_f \times \vec{H}$$



STEP 1

$$\Delta\varphi(x) = 0 \qquad \forall x \in \Omega$$

$$\oint_{\Gamma} G(x,y)\frac{\partial\varphi}{\partial n_x}(x)\, d\Gamma(x) - \oint_{\Gamma} \varphi(x)\frac{\partial G}{\partial n_x}(x,y)\, d\Gamma(x) = \frac{\Theta(y)}{4\pi}\varphi(y) \qquad y \in \Gamma$$

$$\vec{J}_f = -\sigma\nabla\varphi$$

STEP 2

$$\vec{H} = \vec{H}^J + \vec{H}^M \qquad \vec{H}^J -> Biot - Savart$$

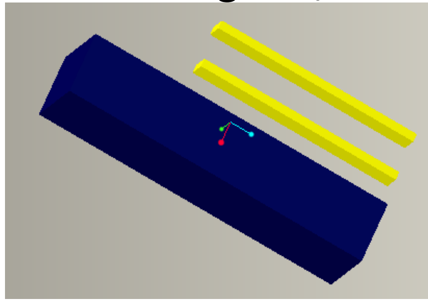
$$\vec{H}^M(x) = \frac{1}{4\pi}\oint_{\Gamma} \sigma(x)\frac{\vec{r}_{xy}}{r_{xy}^3}\, d\Gamma_y + \frac{1}{4\pi}\int_{\Omega} \rho(k)\frac{\vec{r}_{xk}}{r_{xk}^3}\, d\Omega$$

$$\sigma(x) - \frac{\lambda(x)}{2\pi}\oint_{\Gamma} \sigma(y)\frac{\vec{r}_{xy}\cdot\vec{n}}{r_{xy}^3}\, d\Gamma = 2\lambda(x)\vec{H}\cdot\vec{n} - \frac{\lambda(x)}{2\pi}\oint_{\Omega} \rho(k)\frac{\vec{r}_{xk}}{r_{xk}^3}\, d\Omega$$

- Contribution from the non-linear part does not influence the matrix size!!!
- It is just a correction of the matrix RHS.

DC Forces on current-carrying conductors

Example 1 : Modeling of two bus-bars in presence of magnetic material using Pro/E-POLOPT integrated tool



Two bus-bars only

FORCE VECTOR(S) F[N] ON 2 BODIES

No.	Dom.	X-comp. [N]	Y-comp. [N]	Z-comp. [N]	Module
1	0	7.907E+05	-2.932E+02	-1.970E+01	7.907E+05
2	0	-7.916E+05	-2.104E+02	-6.138E+00	7.916E+05

Two bus-bars in presence of linear material

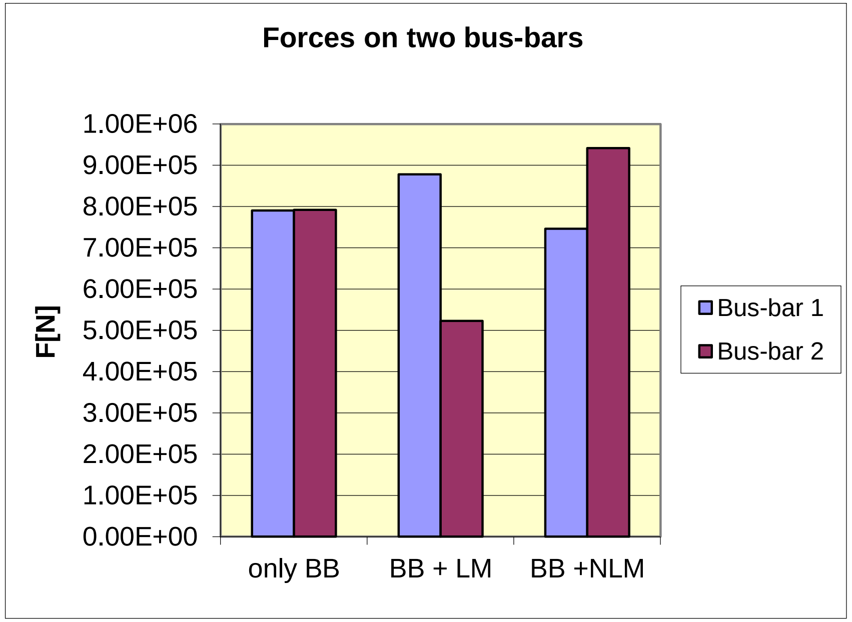
FORCE VECTOR(S) F[N] ON 2 BODIES

No.	Dom.	X-comp. [N]	Y-comp. [N]	Z-comp. [N]	Module
1	0	8.783E+05	1.271E+04	-2.058E+01	8.784E+05
2	0	-5.169E+05	8.003E+04	-8.790E+00	5.230E+05

Two bus-bars in presence of non- linear material

FORCE VECTOR(S) F[N] ON 2 BODIES

No.	Dom.	X-comp. [N]	Y-comp. [N]	Z-comp. [N]	Module
1	0	7.464E+05	-1.668E+04	-1.924E+01	7.465E+05
2	0	-9.391E+05	-6.631E+04	-4.714E+00	9.414E+05



Impact of linear / nonlinear materials on the forces

AC Forces

AC Forces on Current-carrying Conductors

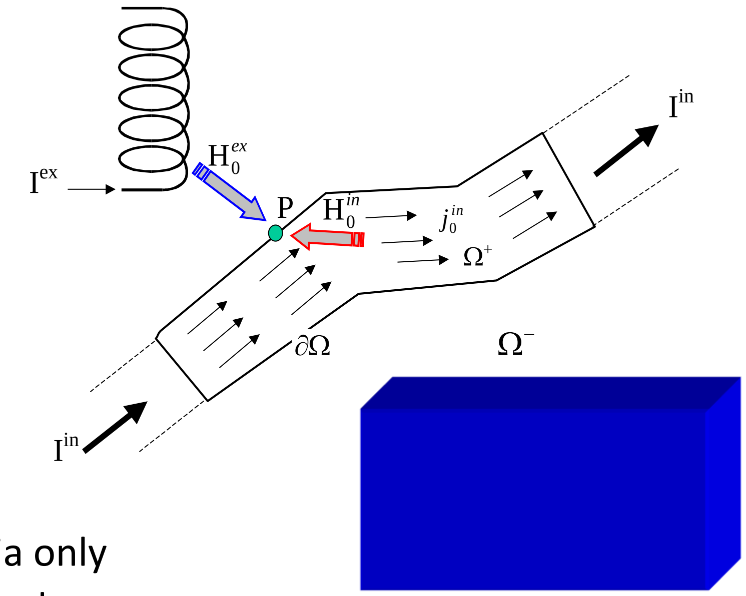
Time –average Lorentz force density : $\bar{f}$

$$\bar{f} = \frac{1}{2}\mathrm{Re}\{\dot{J} \times \dot{B}^{*}\}$$

Key challenge

Calculation of eddy-current $\dot{J}$ taking into account:

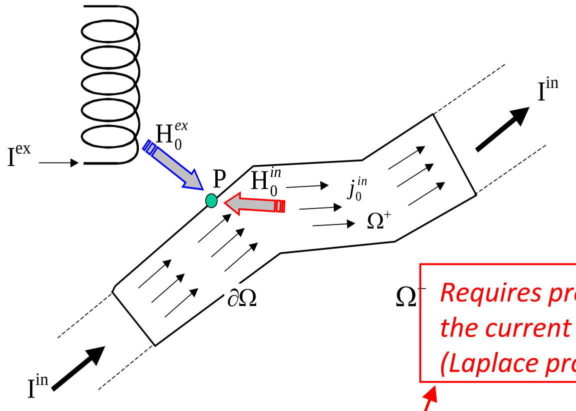
- skin effect
- proximity effect
- presence of the magnetic materials



Approach:

- BEM -> meshing interface between different media only
- New <u>innovated</u> formulation for skin-effect treatment
- New Galerkin integration technique (geometrical singularities)
- New compression / acceleration techniques used

Skin-effect treatment (cont.)



Requires pre-calculation of the current distribution (Laplace problem)

$$\mathbf{H}^{out} = \mathbf{H}^- = -\nabla\varphi + \mathbf{H}_0, \qquad \mathbf{H}_0(\mathbf{r}) = \mathbf{H}_{0_P} = \mathbf{H}_{0_P}^{in} + \mathbf{H}_{0_P}^{ex}$$

$$\mathbf{H}_0^{in}(\mathbf{r}) = \int_{\mathbf{R}^3} \frac{\mathbf{j}_0^{in}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{4\pi \cdot \|\mathbf{r}-\mathbf{r}'\|^3} d\mathbf{r}' \quad \forall \mathbf{r} \in \mathbf{R}^3.$$

Boundary value problem

$$rot\, rot\, \mathbf{H}^+ + i\omega\sigma\mu \cdot \mathbf{H}^+ = 0 \quad \forall \mathbf{r} \in \Omega^+,$$

$$\Delta\varphi = 0 \quad \forall \mathbf{r} \in \Omega^-,$$

$$\mathbf{n} \times \left(\mathbf{H}^+ + \nabla\varphi\right) = \mathbf{n} \times \mathbf{H}_0 \quad \forall \mathbf{r} \in \partial\Omega,$$

$$\mathbf{n} \cdot \left(\mu\mathbf{H}^+ + \mu_0\nabla\varphi\right) = \mu_0 \cdot \mathbf{n} \cdot \mathbf{H}_0 \quad \forall \mathbf{r} \in \partial\Omega.$$

Integral representation

$$\mathbf{H}^+(\mathbf{r}) = rot\left(\frac{1}{4\pi} \oint_{\partial\Omega} \mathbf{j}(\mathbf{r}') \frac{e^{-(1+i)k\|\mathbf{r}-\mathbf{r}'\|}}{\|\mathbf{r}-\mathbf{r}'\|} dS_{\mathbf{r}'}\right); \quad \mathbf{r} \in \Omega^+$$

$$\phi^-(\mathbf{r}) = \frac{1}{4\pi} \oint_{\partial\Omega} \frac{\sigma^m(\mathbf{r}')}{\|\mathbf{r}-\mathbf{r}'\|} dS_{\mathbf{r}'}; \quad \mathbf{r} \in \Omega^-$$

$H-\varphi$ formulation

Equivalent currents

$$\frac{1}{2}\mathbf{j}(\mathbf{r}) + \frac{1}{4\pi} \oint_{\partial\Omega} \mathbf{n}(\mathbf{r}) \times (\mathbf{j}(\mathbf{r}') \times \nabla(\frac{e^{-(j+i)k\|\mathbf{r}-\mathbf{r}'\|}}{\|\mathbf{r}-\mathbf{r}'\|}))dS_{\mathbf{r}'} - \frac{1}{4\pi} \oint_{\partial\Omega} \sigma^m(\mathbf{r}')\mathbf{n}(\mathbf{r}) \times \nabla(\frac{1}{\|\mathbf{r}-\mathbf{r}'\|})dS_{\mathbf{r}'} = -\mathbf{n}(\mathbf{r}) \times \mathbf{H}_0(\mathbf{r})$$

Equivalent charges

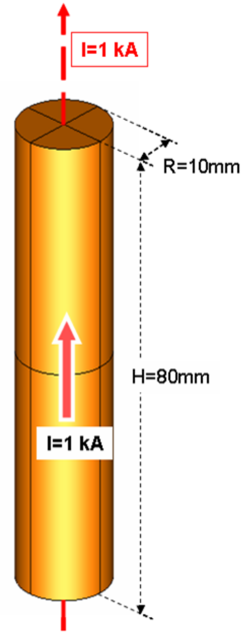
$$\frac{1}{2}\sigma^m(\mathbf{r}) + \frac{1}{4\pi} \oint_{\partial\Omega} \sigma^m(\mathbf{r}')\mathbf{n}(\mathbf{r}) \cdot \nabla(\frac{1}{\|\mathbf{r}-\mathbf{r}'\|})dS_{\mathbf{r}'} + \frac{\mu}{4\pi\mu_0} \oint_{\partial\Omega} \mathbf{n}(\mathbf{r}) \cdot (\mathbf{j}(\mathbf{r}') \times \nabla(\frac{e^{-(1+i)k\|\mathbf{r}-\mathbf{r}'\|}}{\|\mathbf{r}-\mathbf{r}'\|}))dS_{\mathbf{r}'} = -\mathbf{n}(\mathbf{r}) \cdot \mathbf{H}_0(\mathbf{r})$$

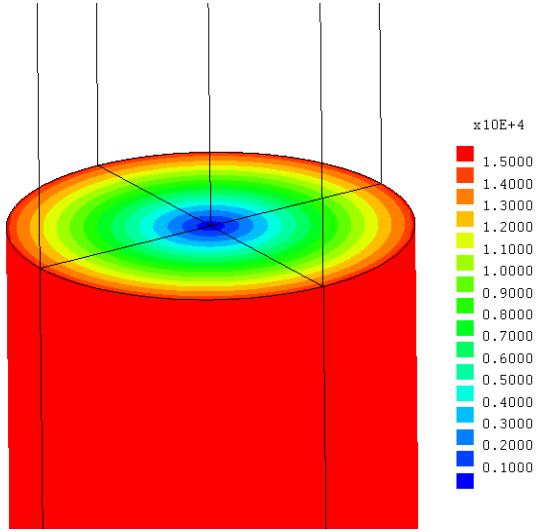
$$\vec{n} \times H_i = \vec{n} \times H_a$$
$$\vec{n} \cdot B_i = \vec{n} \cdot B_a$$

$$\begin{bmatrix} A_1 & B_1 \\ B_2 & A_2 \end{bmatrix} \begin{pmatrix} j \\ \sigma^m \end{pmatrix} = \begin{pmatrix} -2\mathrm{n} \times \mathrm{H}_0 \\ -2\mathrm{n} \cdot \mathrm{H}_0 \end{pmatrix}$$

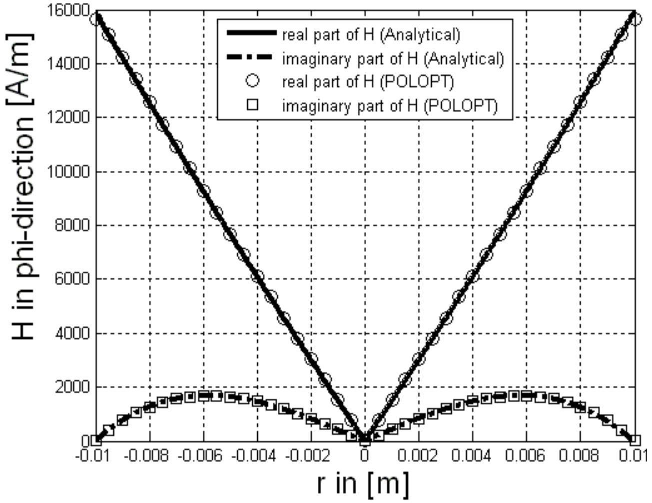
$$K(\xi,\eta) = \frac{1}{4\pi r} e^{-\beta r} \qquad G(\xi,\eta) = \frac{1}{4\pi r}$$

Example 1: Skin-effect in Cu/Fe conductors

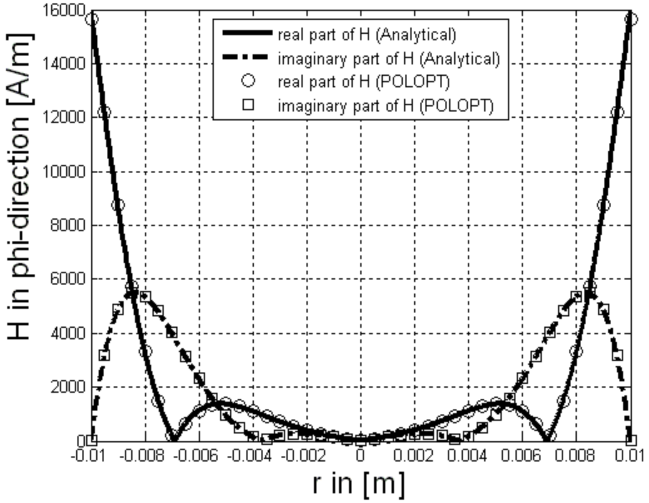




Field penetration in the Cu-conductor



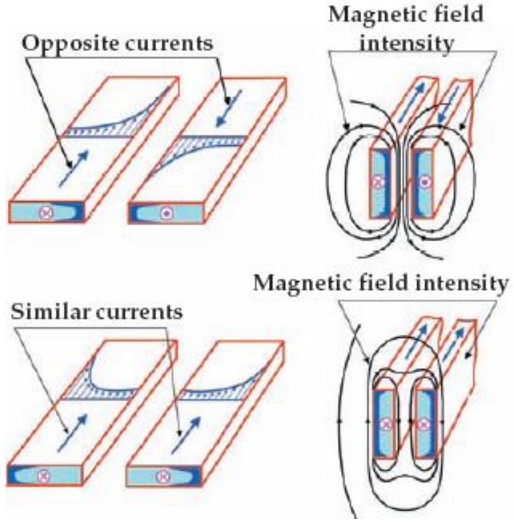
m = 1

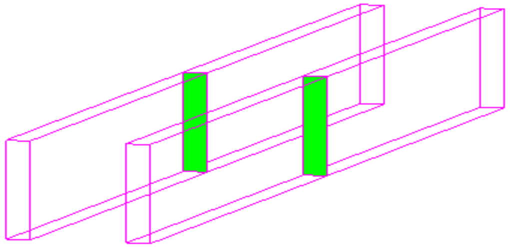


m = 200

Example 2: Proximity effect in parallel bus-bars

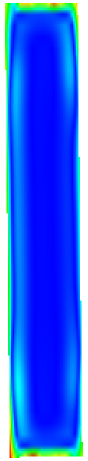
Proximity effect



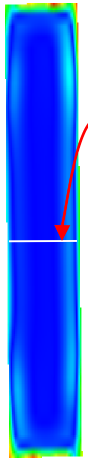


Copper

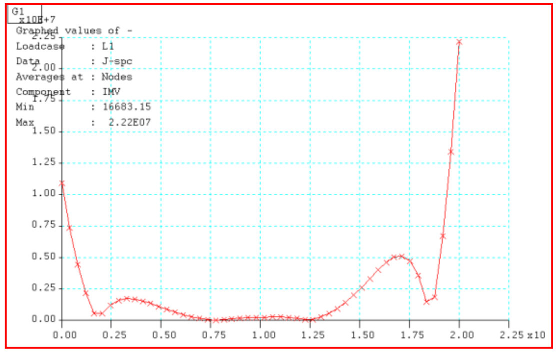
Current in same direction



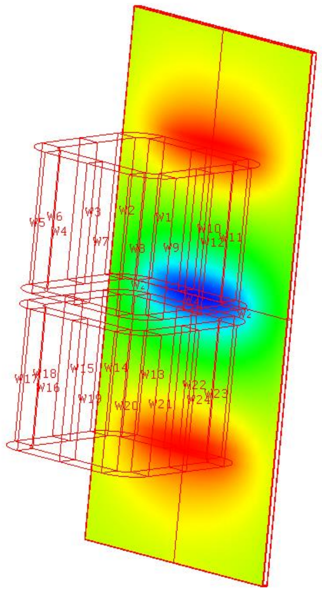
Steel-200

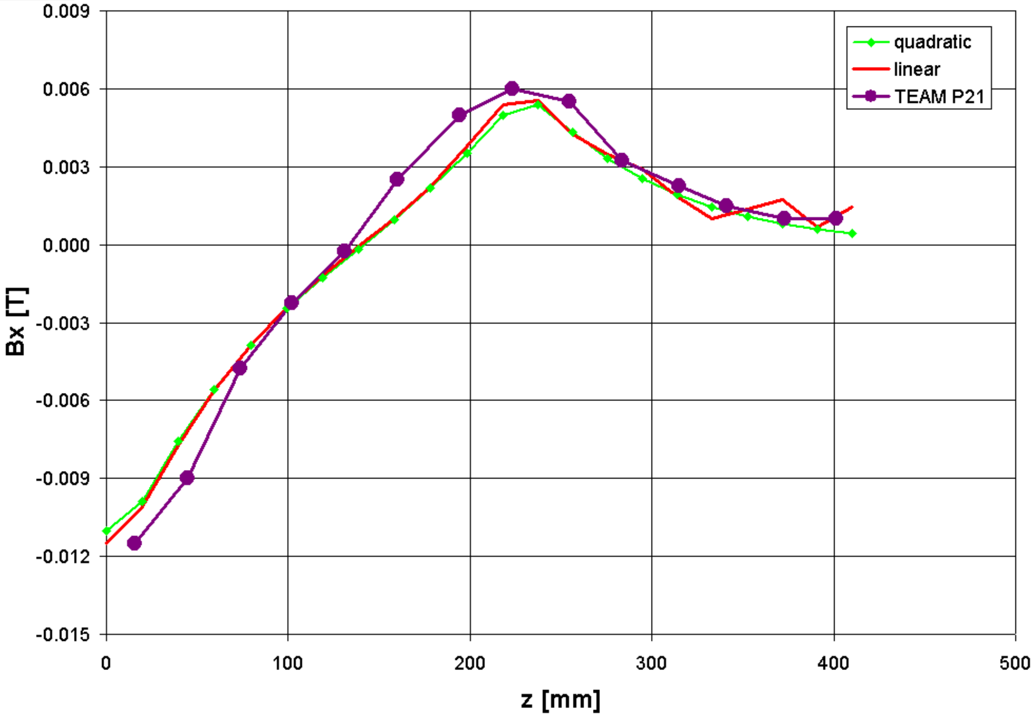


Current (real part) along the graph line



Example 2: Validation TEAM P21





- B_X in z-direction
- non-magnetic steel

  μ=1

Forces on Ferromagnetic Structure

Forces on magnetic bodies

Force density in incompressible linear magnetizable media

$$\vec{f} = \mu_0 \vec{J}_f \times \vec{H} + \mu_0 \vec{M} \cdot \nabla \vec{H}$$

$$\vec{f} = \mu_0 \vec{M} \cdot \nabla \vec{H}$$ → in the absence of the conduction current

Total force

$$\vec{F} = \int_\Omega \vec{f}(r) d\Omega = \oint_\Gamma t d\Gamma$$

t – traction vector of the tensile forces per unit

Maxwell stress method

Magnetic charge method

Magnetization current method

Virtual work method

? best method

- When polarized or magnetized materials are present, the Lorentz force law must be applied not only to the free charges within the materials, i.e. the surface charges and currents discussed earlier, but also to the orbiting and spinning charges bound within atoms.
- When the Lorenz force equation is applied to these bound charges, the result is the Kelvin polarization and magnetization force densities.
- Thus, Kelvin forces must be added to the Lorentz forces on the free charges!

L.D. Landau, E.M. Lifshitz: Electrodynamicsy in Continuous Media, Nauka, Moskow, 1982

Forces on magnetic bodies

Force density in incompressible linear magnetizable media

$$\boldsymbol{f} = \mu_0 \boldsymbol{J}_f \times \boldsymbol{H} + \mu_0 \boldsymbol{M} \cdot \nabla \boldsymbol{H}$$

$$\boldsymbol{f} = \mu_0 \boldsymbol{M} \cdot \nabla \boldsymbol{H}$$ → in the absence of the conduction current

Total force

$$\boldsymbol{F} = \int_\Omega \boldsymbol{f}(r) d\Omega = \oint_\Gamma t d\Gamma$$

t – traction vector of the tensile forces per unit

J.R. Melcher: Continuum Electromechanics, M.I.T Press, Cambridge, MA 1981

$\boxed{\text{Maxwell stress method}}$ ⟶

$$\boldsymbol{F} = \frac{1}{\mu_0} \iint \overline{\boldsymbol{S}} \cdot \boldsymbol{n} dA$$

$$\boldsymbol{S} = \frac{1}{\mu_0} \begin{bmatrix} B_x^2 - B^2/2 & B_x B_y & B_x B_z \\ B_y B_x & B_y^2 - B^2/2 & B_Y B_z \\ B_z B_x & B_z B_y & B_z^2 - B^2/2 \end{bmatrix}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2$$

Magnetic charge method

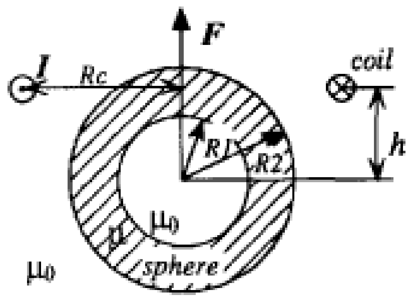
Magnetization current method

Virtual work method

Forces on magnetic bodies

Benchmark Problem: Levitated Sphere (Linear case)

Z. Ren: **Comparison of Different Force Calculation Methods in 3D FEM modeling**, IEEE on Mag, 30, 5, Sep.1994



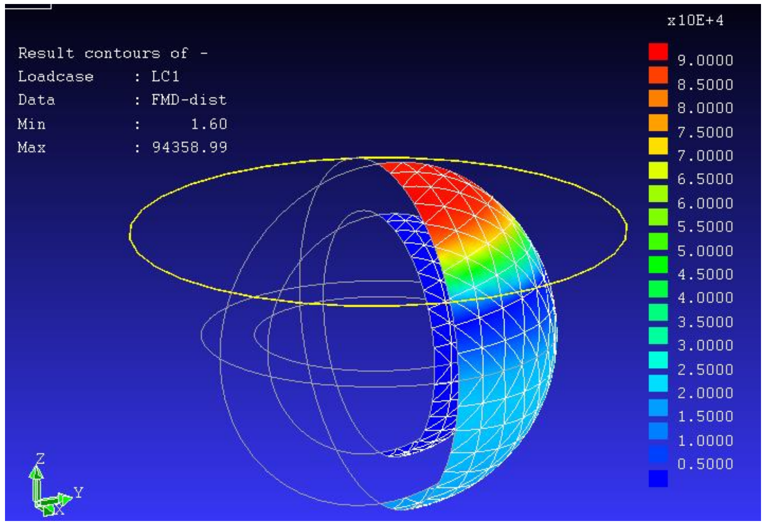
$R1=0.035m$
$R2=0.05m$
$Rc=0.07m$
$h=0.03m$
$\mu r=500$
$I=20000A$

FEM-based methods

Comparison of Levitation Forces (Newton)

methods	ϕ formulation	A formulation	two dual formulations
F_{Mst}	292.75	351.08	351.23
F_{Jm}	88.75	293.87	366.26
F_{Qm}	292.75	351.08	351.23
F_{Sfd}	292.35	350.97	351.26
F_{ε}	0.408	0.114	-0.03
virtual work	362.46	353.82	
analytical	372.88		

$$\varepsilon \approx 2-5\%$$



Force density distribution

POLOPT

FORCE VECTOR(S) F[N] ON 1 BODIES

No.	Dom.	X-comp. [N]	Y-comp. [N]	Z-comp. [N]	Module
1	2	0.000E+00	0.000E+00	3.716E+02	3.716E+02

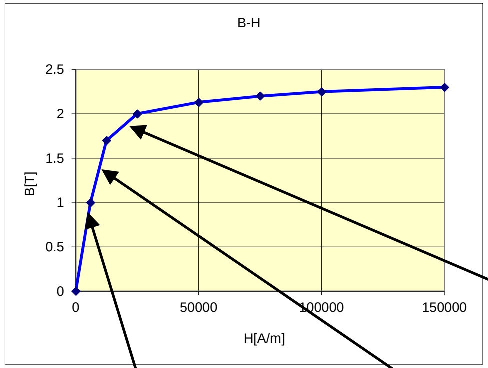
$$\varepsilon = 0.34\%$$

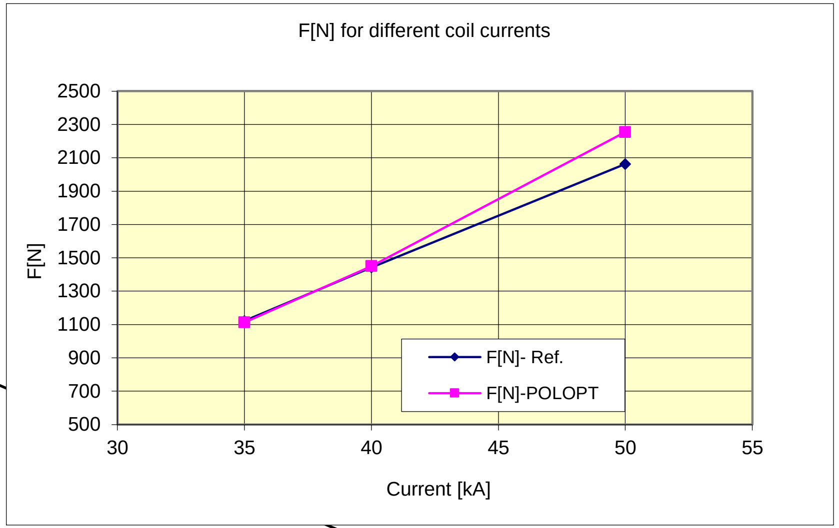
Forces on magnetic bodies

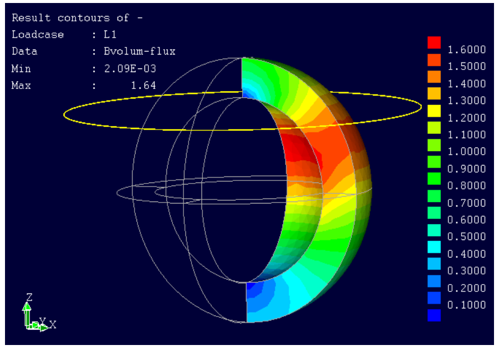
Benchmark Problem: Levitated Sphere (Non-linear case)

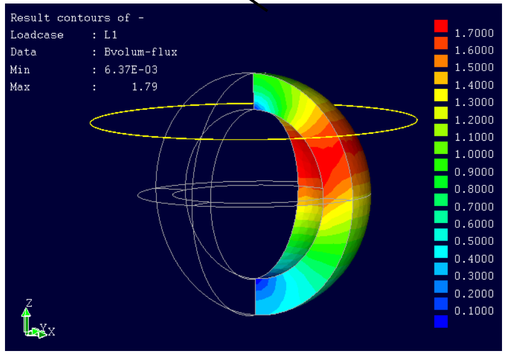
[1] S. Bobbio at all: **Equivalent Source Methods for the Numerical Evaluation of Magnetic Force with Extension to Nonlinear Materials**, IEEE Tran. on Mag., Vol. 36, No. 4, July 2000

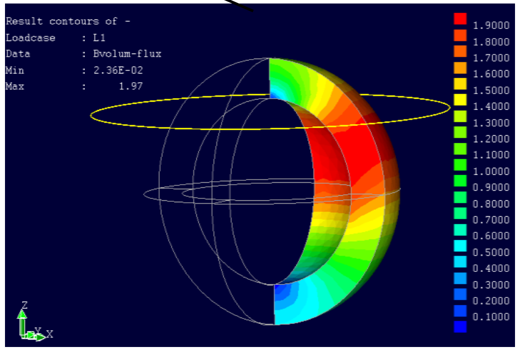
B-H curve for the nonlinear test problem





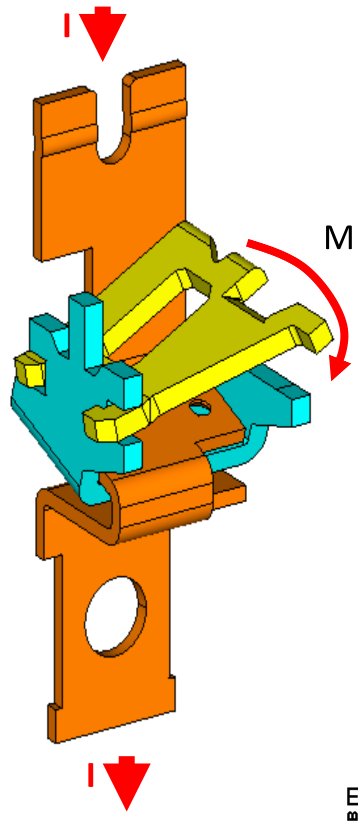


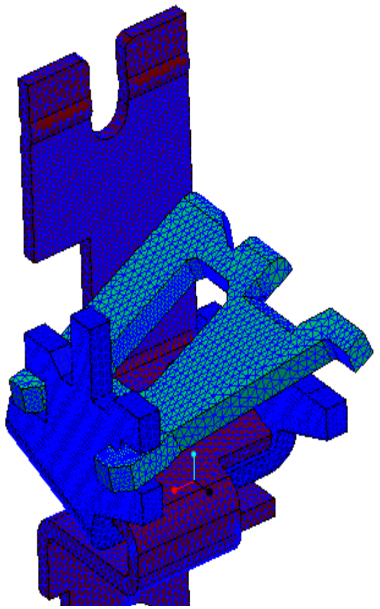




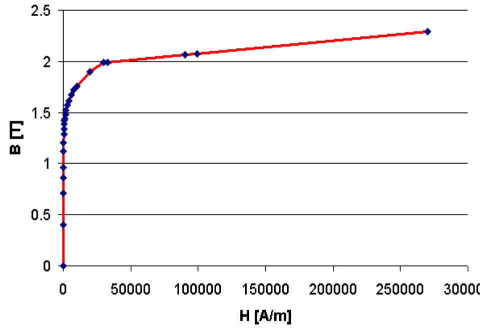
Forces on magnetic bodies

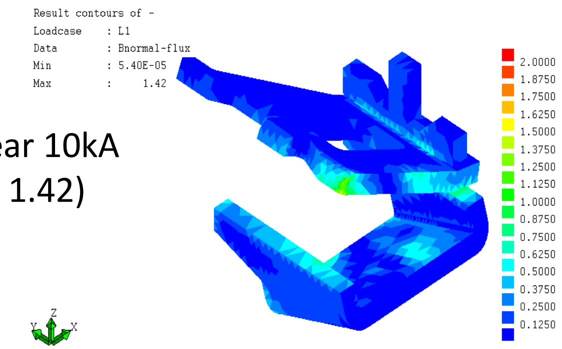
Low-voltage switch, Sace, Italy



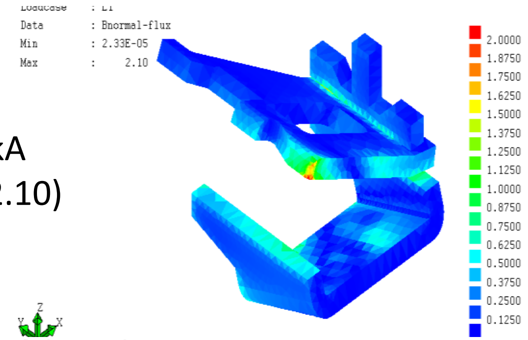


B-H curve

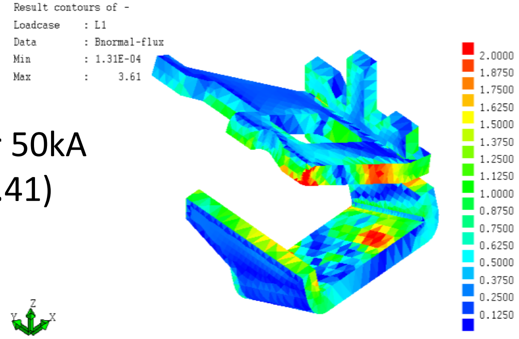




non-linear 10kA
(Bmax = 1.42)
F = 39N



linear 10kA
(Bmax = 2.10)
F = 58N

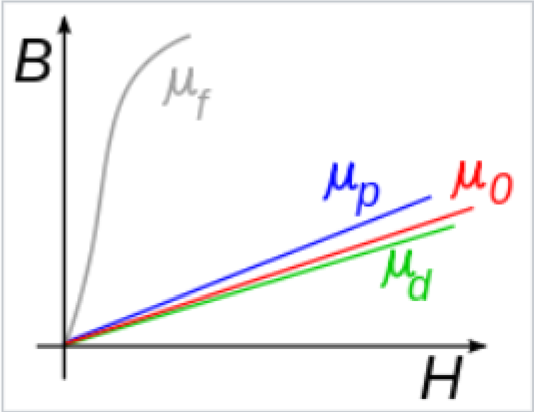


non-linear 50kA
(Bmax = 3.41)
F = 393N

Forces on Non-magnetic Structures

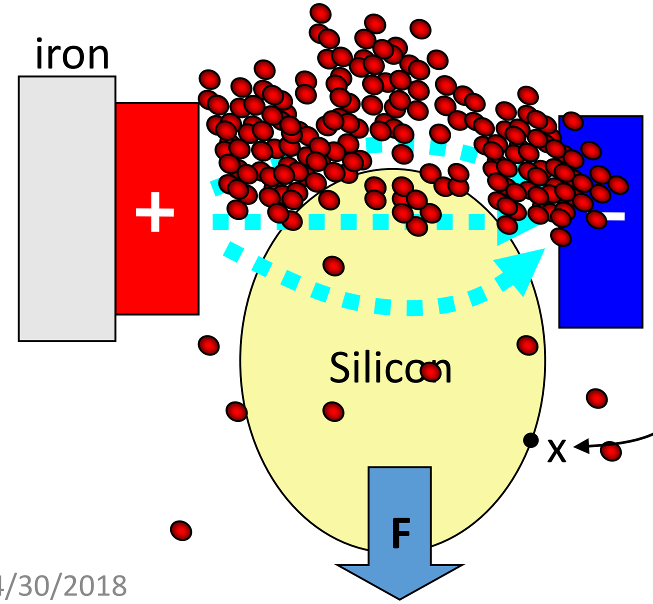
Forces on non - magnetic bodies

- **Ferromagnetic** ($\mu >> 1; \chi >> 0$): attractive effect, (iron, ferromagnetic alloys,…)
- **Paramagnetic** ($\mu > 1; \chi > 0$): linear attractive effect, no remanent magnetism, (ferro-fluids, oxygen, aluminum,…)
- **Diamagnetic** ($\mu < 1; \chi < 0$): repelled effect (bismuth, wood, copper, gold, superconductors, …)



Simplified comparison of permeabilities for: ferromagnets (μ_f), paramagnets (μ_p), free space (μ_0) and diamagnets (μ_d)

Example:

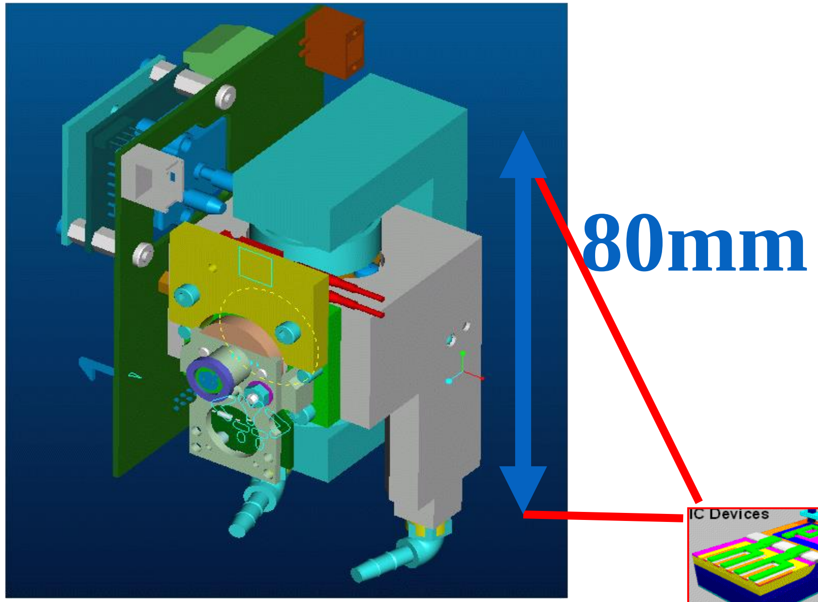


$$\vec{H}(x) = \frac{1}{4\pi}\int_S \sigma(x)\frac{\vec{r}_{x\varsigma}}{r^3_{x\varsigma}}dS_x + \frac{1}{4\pi}\int_V \rho(N)\frac{\vec{r}_{xN}}{r^3_{xN}}dV_N + \vec{H}^0(x)$$

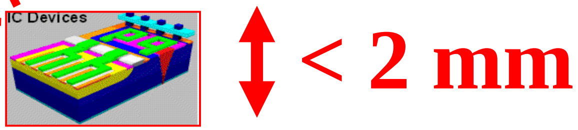
$$\vec{F}(x) = -0.5\int \mu_0 \Delta\chi H^2 d\vec{S}$$

MEMS Simulation O_2 sensor- layout

Goal: Achieve IC layout having at least the same performances as the standard layout!



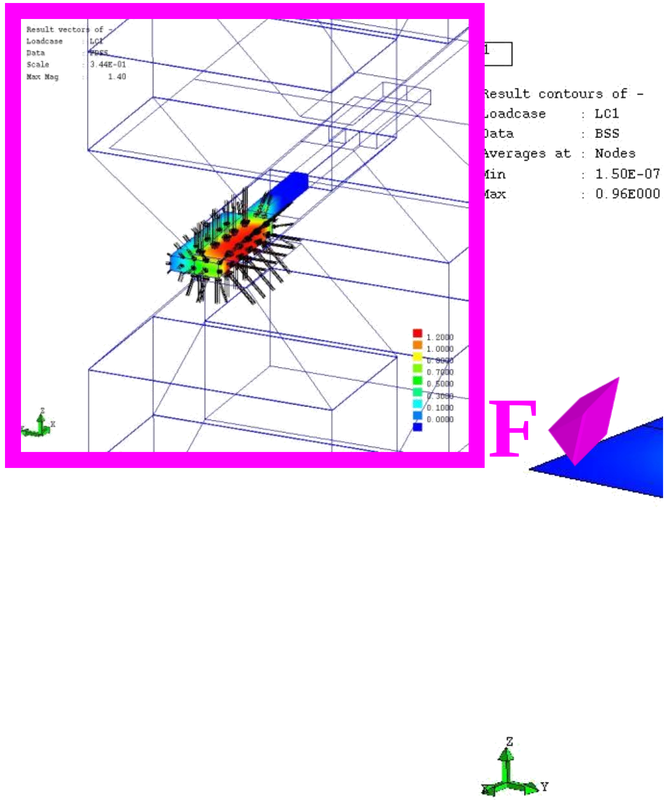
Classical layout

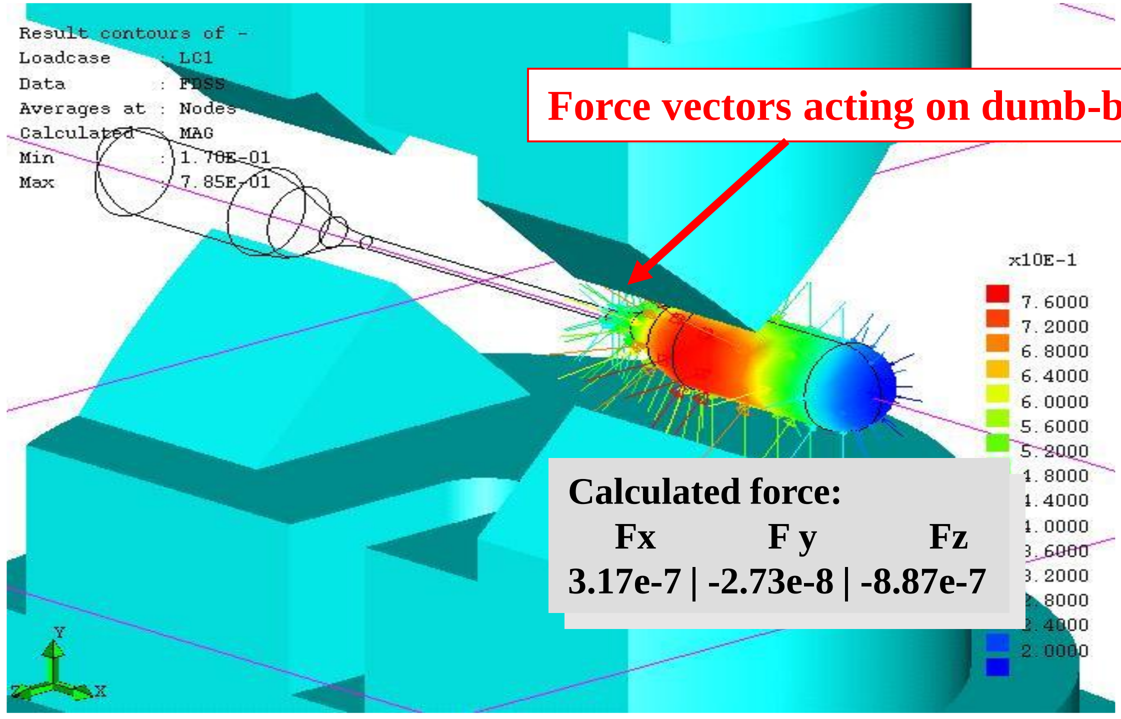


IC layout

MEMS Simulation O_2 sensor- layout

Force density distribution





Deviation to measurement: < 10%

Coupled Electromagnetics- Molecular Dynamics - Mechanics simulation