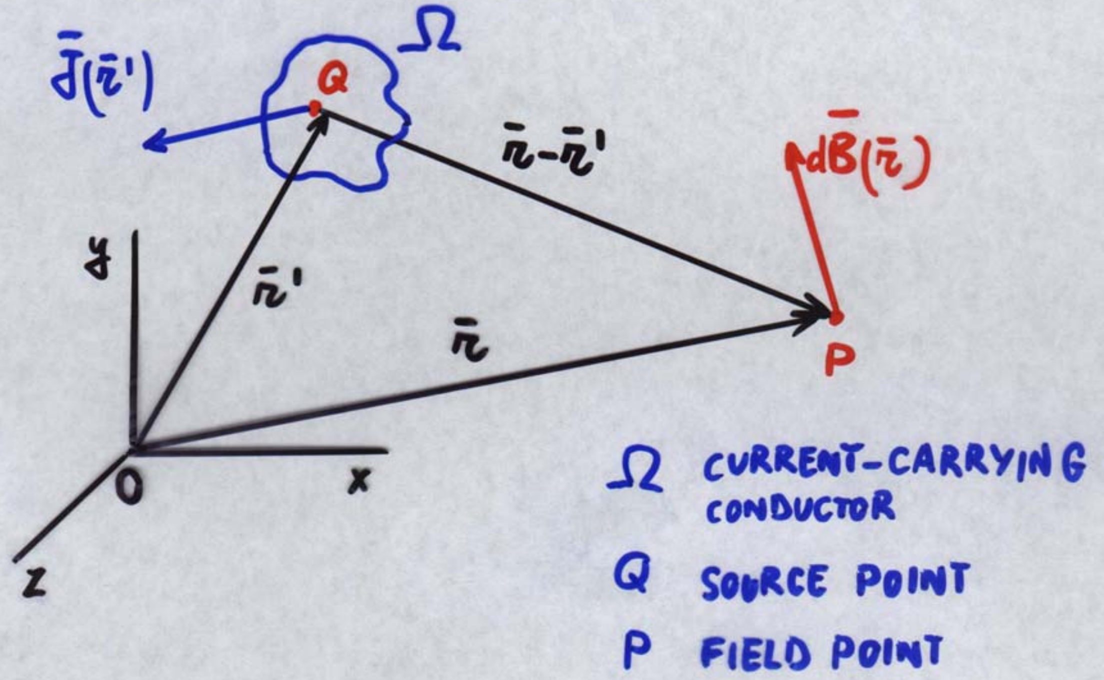
Laplace's law of magnetostatic field

MAGNETIC FIELD

IN A LINEAR, HOMOGENEOUS, ISOTROPIC MATERIAL

(UNBOUNDED DOMAIN)



LAPLACE LAW

$$d\bar{B}(\bar{r}) = \frac{\mu_0}{4\pi} \frac{\bar{J}(\bar{r}') \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} \, d\Omega$$

$$\bar{B} = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\bar{J} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} \, d\Omega$$

$$\Longrightarrow \quad \bar{B} \perp \bar{J} \;, \quad \bar{B} \perp (\bar{r} - \bar{r}')$$

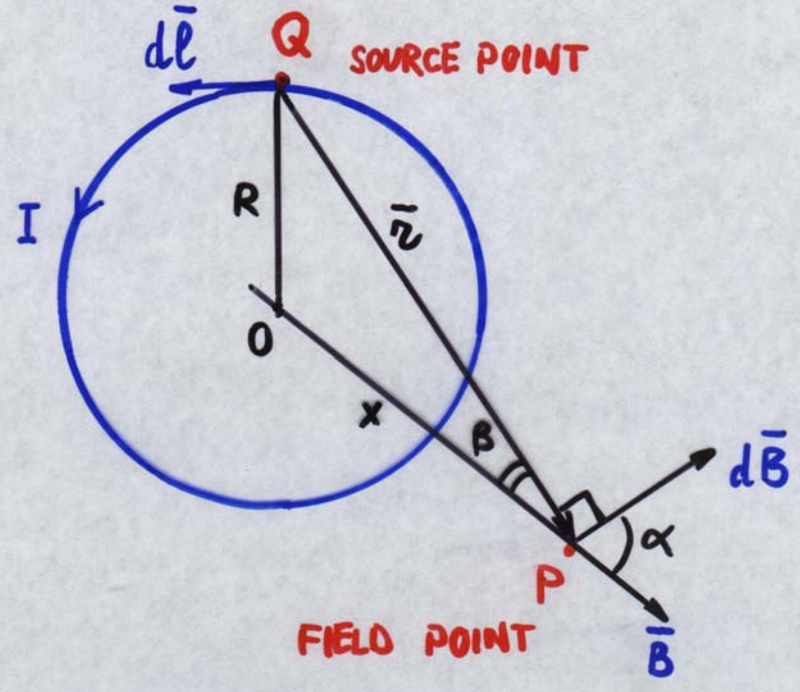
LAPLACE LAW (CONT.D)

FOR A THIN CONDUCTING WIRE

$d\Omega = S\, dl$ — LENGTH (S: CROSS SECTION) , $J\, d\Omega = \frac{I}{S} S\, dl$

$$\Rightarrow \quad \bar{B} = \frac{\mu_0 I}{4\pi} \oint_l \frac{d\bar{l} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

MAGNETIC FIELD ALONG THE AXIS OF A CIRCULAR COIL



$$d\bar{B}(P) = \frac{\mu_0 I}{4\pi} \frac{d\bar{\ell} \times \bar{r}}{|\bar{r}|^3}$$

CIRCULAR COIL (CONT.D)

$$dB(P) = \frac{\mu_0 I \, dl}{4\pi r^2}$$

$$dB_x = dB \cos\alpha = dB \sin\beta = dB \frac{R}{\sqrt{R^2 + x^2}}$$

$$dB_x = \frac{\mu_0 I R \, dl}{4\pi (R^2 + x^2)^{3/2}} \quad , \quad B_x = \oint_l dB_x$$

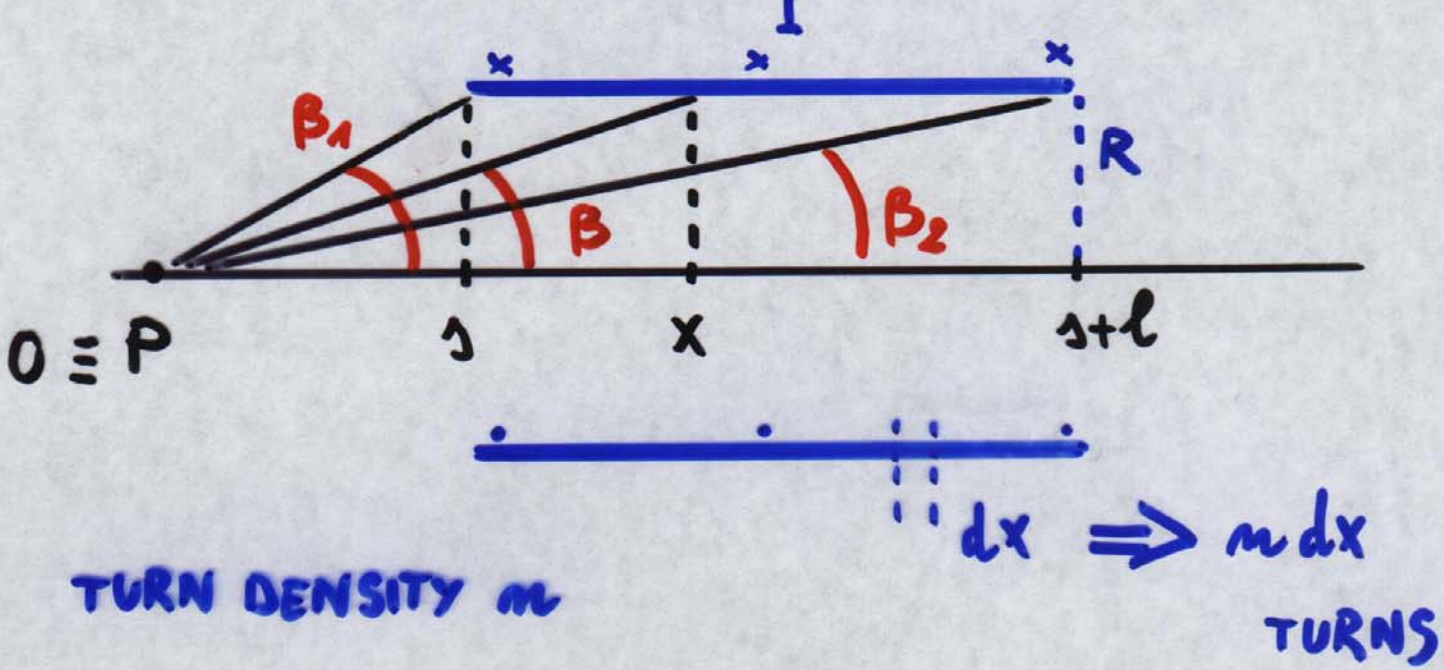
CIRCULAR COIL (CONT.D)

$$B(x) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+x^2)^{3/2}}$$

$$x=0, \quad B = \frac{\mu_0 I}{2R}$$

$$x \gg R, \quad B \approx \frac{\mu_0 I}{2} \frac{R^2}{x^3}$$

MAGNETIC FIELD ON THE AXIS OF A SOLENOID OF FINITE LENGTH ℓ



TURN DENSITY n

FINITE-LENGTH SOLENOID (CONT.D)

$$dB(P) = \frac{\mu_0 n\, dx\, I}{2} \frac{R^2}{(R^2+x^2)^{3/2}}$$

$$\cos\beta = \frac{x}{\sqrt{x^2+R^2}}$$, β INTEGRATION VARIABLE

$$-\sin\beta\, d\beta = \frac{R^2\, dx}{(R^2+x^2)^{3/2}}$$

FINITE-LENGTH SOLENOID (CONT.D)

$$dB(P) = -\frac{\mu_0 n I}{2} \sin\beta \, d\beta$$

$$B(P) = \int_{\beta_1}^{\beta_2} dB = \frac{\mu_0 n I}{2} \left(\cos\beta_2 - \cos\beta_1\right)$$

$$\cos\beta_2 = \frac{s+l}{\sqrt{(s+l)^2 + R^2}} \quad , \quad \cos\beta_1 = \frac{s}{\sqrt{s^2 + R^2}}$$