Software Methods and Tools for the Design and Optimization of Electromechanical Devices

Lecture 3: The Design Process and the Virtual Laboratory

Search – Explore and Exploit

Outline

- Design and Diagnostics
- The Virtual World and Reality
- Hierarchical Design
- Optimization Issues
- Sensitivity of Performance
- Multiple Design Issues

Design and Diagnostics - Inverse Problems

- Frequently, in working with electromagnetic devices, the questions that are asked are:
  - What form of device can achieve the results that are needed?
    - E.g. the driving profile of an electric vehicle
  - What has happened to the system to modify its behaviour?
    - Fault identification in a device
    - Reconstruction of an anomaly – a crack in a pipe, a tumour in a biological system

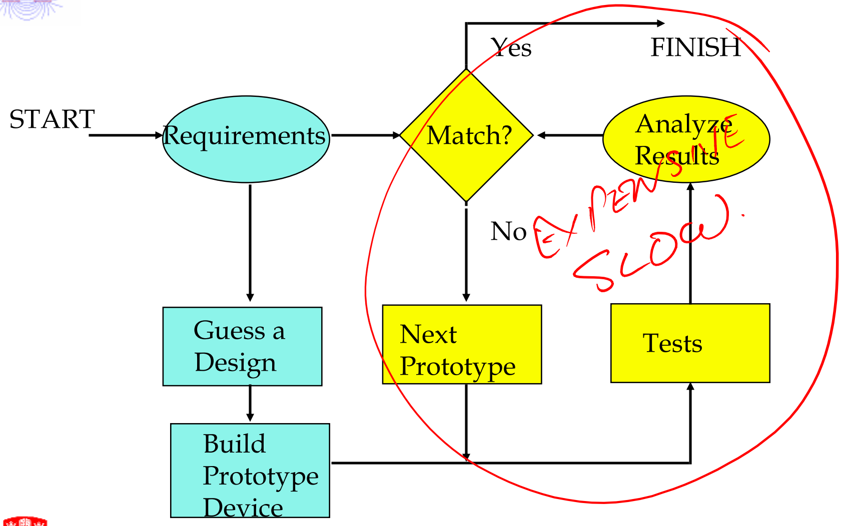
Design and Diagnosis

- In design, the goals can vary:
  - Find the best design possible
  - Modify and existing design to improve it in some way
- Typical design goals:
  - Minimize cost
  - Generate a specific force
  - Exhibit a particular terminal impedance
  - …

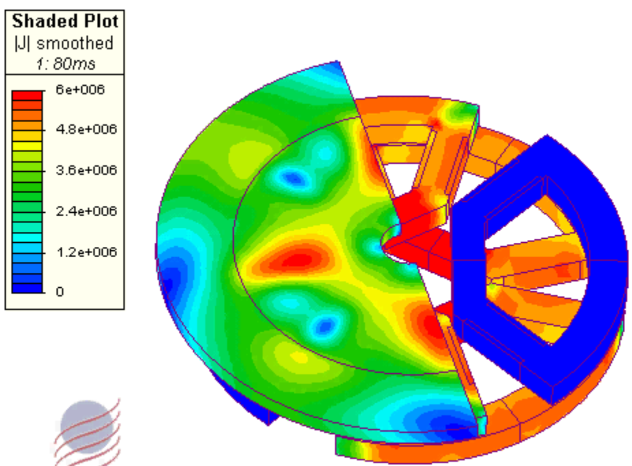
The Inverse Problem

- Inverse problems are characterized by:
  - Small input vectors – large output vectors
  - Non-unique solutions
- In general, the problem is backwards..
  - E.g. forces are created as a result of an electromagnetic field but the question is
    - what field is needed to create a given force?

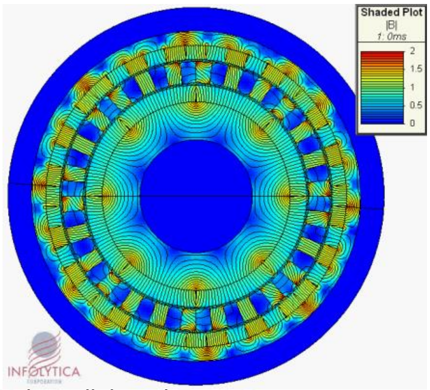
The Design Process

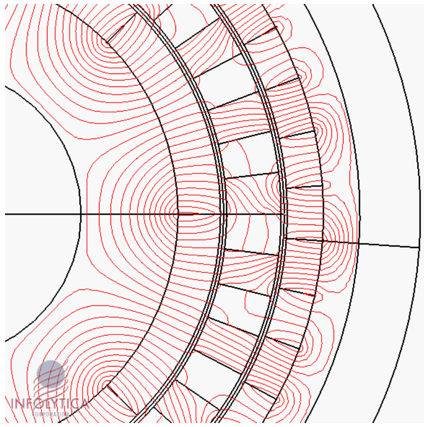


Disk rotor induction motor



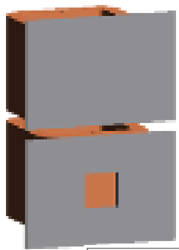
Magnetic gear – Sheffield U magnetic - dynamic

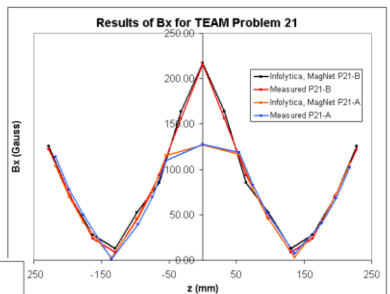


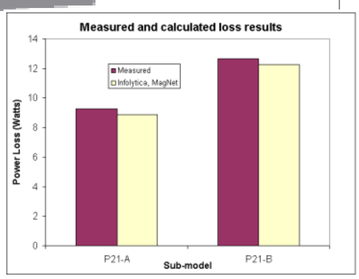


- Kais Atallah and Dave Howe
- *IEEE Trans Mag*, Vol. **37**, No. 4, pp. 2844-2846, July 2001.

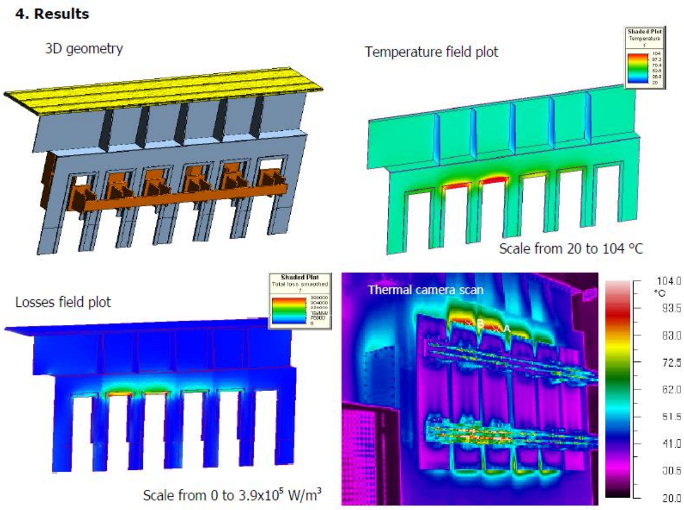
Team 21 A Shielding Problem







50MVA Transformer Tank



Creating the Virtual Laboratory

- Models provide the virtual laboratory
- The accuracy of the model determines when physical structures are needed
- Different levels of "virtuality" needed – speed vs accuracy

Two Branches of Development

- Modelling has advanced due to
  - Increases in computational capability
  - Developments in numerical analysis
  - Improvements in software tools
- Tools have been developed to
  - Provide fast "sizing"
  - Compute "Ballpark" estimates of performance
  - Generate designs with a reasonable confidence level

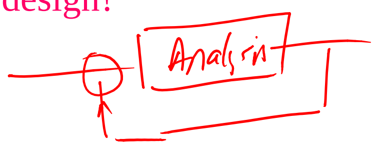
The Virtual World and Reality

- Although we can solve problems to high degrees of accuracy. What does this mean?
  - We solve the non-linear equations set
  - The answers are only as good as the representations of the equations – if they are wrong we get a highly accurate answer to the wrong problem.
  - How do we know that the solution models reality?

The Virtual World and Reality

- All these results come from the creation of sophisticated *analysis tools*
  - *The solution to the forward problem*
  - The advances have been impressive but

What about design?

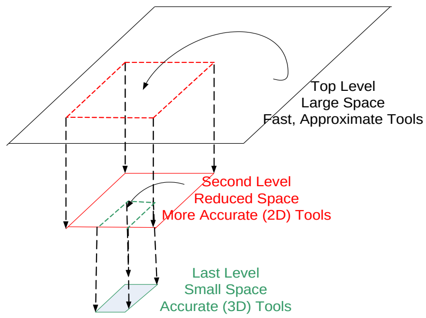


Design Needs

- Analytical models which reduce the need for prototyping
- Fast and approximate – ballpark – models to begin the process
- More accurate models as the process develops
- When good enough - prototype

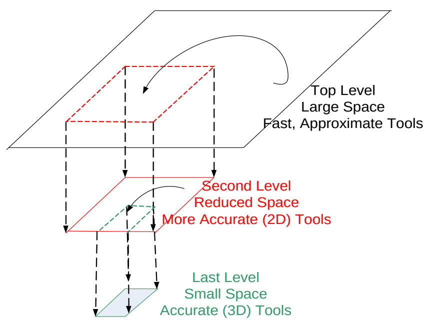
The Design Process

- The design process is one of narrowing down the search space by means of a hierarchical, iterative process…
- At each level, the space is "explored" and "exploited"



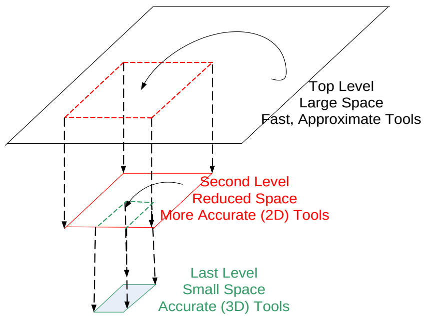
The Design Process

- At the top level, the model needed has to be fast and approximate –
- At the next level, more complex models – fast but more accuracy



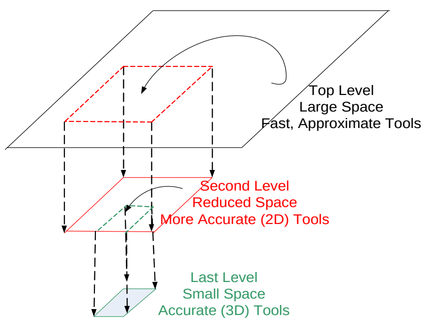
The Design Process

- At the third level, if necessary, a 2-D field solution
- Finally, if required, slow and complete 3-D field analysis *for high accuracy*



The Design Process

- The issue:
  - *What level of accuracy is needed before moving to a physical prototype?*
- Examine the requirements and the tools



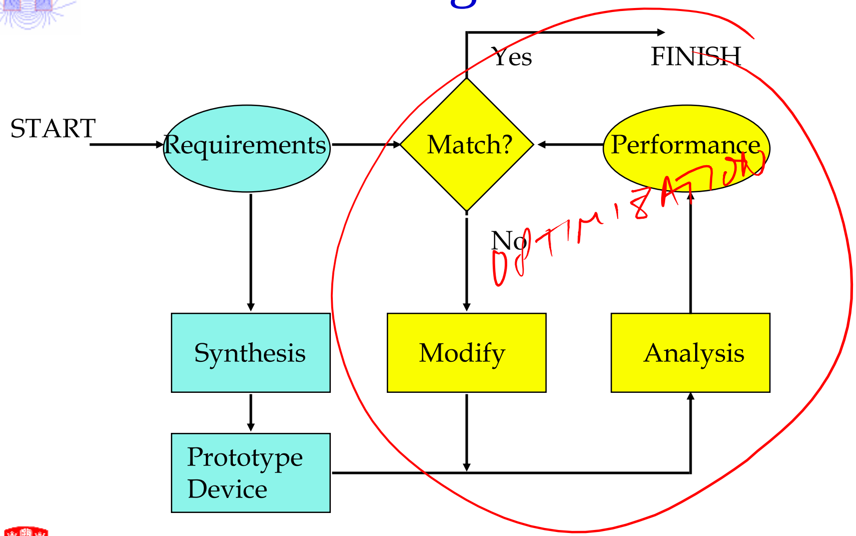
The Design Process

- So within the Design process, analysis tools are
  - Needed to verify a design
  - Needed to indicate how to modify it
- The design process moves through the analysis hierarchy (circuits, 2d, 3d) *until the design solution is "good enough"…*

The Design Process

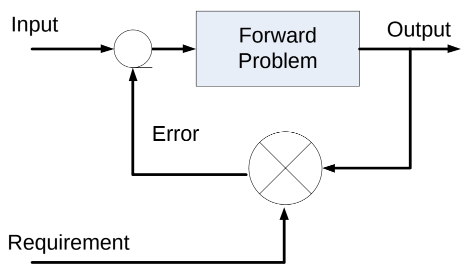
- At each level of the process, a search for the "best" solution takes place
- The space being modelled at the level is explored and a decision is made on where and when to move to a new level – exploit the knowledge.
- The search process is commonly referred to as "**optimization**"

The Design Process



The Goal of Optimization

- To find an output parameter set which matches a pre-specified set as closely as possible:



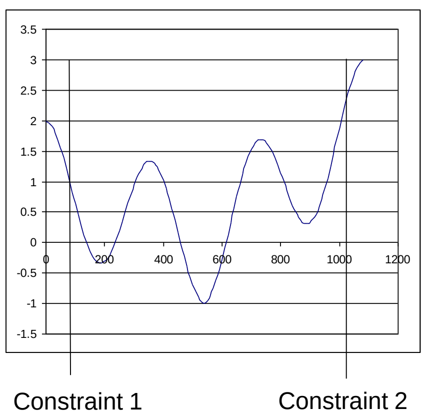
Requirements for Optimization

- Optimization is a process of searching an often unknown space for a solution which meets a set of performance criteria.
- Through iteration, the process "learns" as it proceeds
  - Three questions:
    - What do we mean by "optimization"
    - How complex is the design (cost function) space?
    - Can we exploit this learning?

COST, FORCE EFFICIENCY

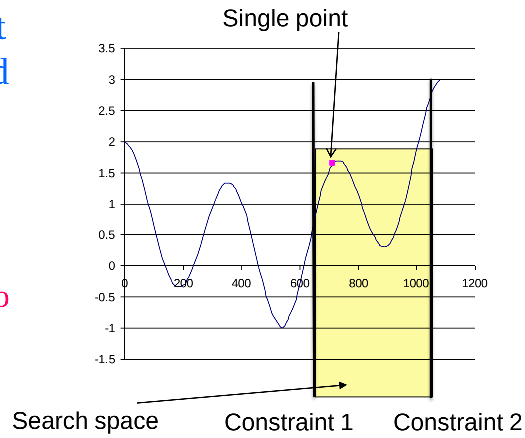
Requirements for Optimization

- So – Issues:
  - The "shape" of the space of solutions is not known
  - There may be more than one "optimal" solution
    - Local Minima
    - Global Minimum
  - Constraints can affect the search



Requirements for Optimization

- Choose a single point inside the constrained area..
  - Based on a previous solution?
  - Now what?
  - Move in a direction to decrease function
    - Needs gradient



Requirements for Optimization

- Consequently, the desired optimization process has two components:
  - Exploration
    - where is the solution likely to be?
  - Exploitation
    - given a likely location is there an appropriate minimum there?
- How can we do this?

Device Optimization - Explore

- *Exploration* requires evaluating solutions at various points in the design space.
- These points can be chosen in a variety of ways:
  - Randomly
    - Guess a possible point
  - According to a set of rules
    - Derived from knowledge of the surface structure

Device Optimization - Exploit

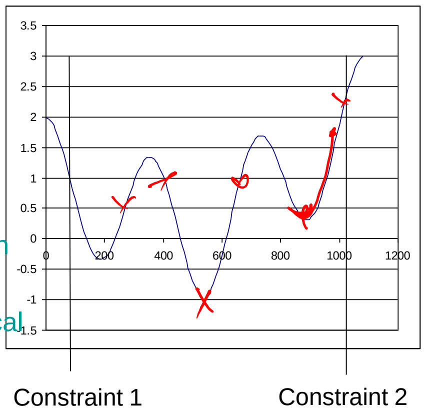
- *Exploitation* uses knowledge gained about a region of the design space to perform a local search – *a local improvement in the objective function.*
- This phase can be performed in several ways
  - Estimate gradients and use a conventional steepest descent algorithm.
  - Just guess and try to use answers to make the next guess better.

Optimization Processes - Exploitation

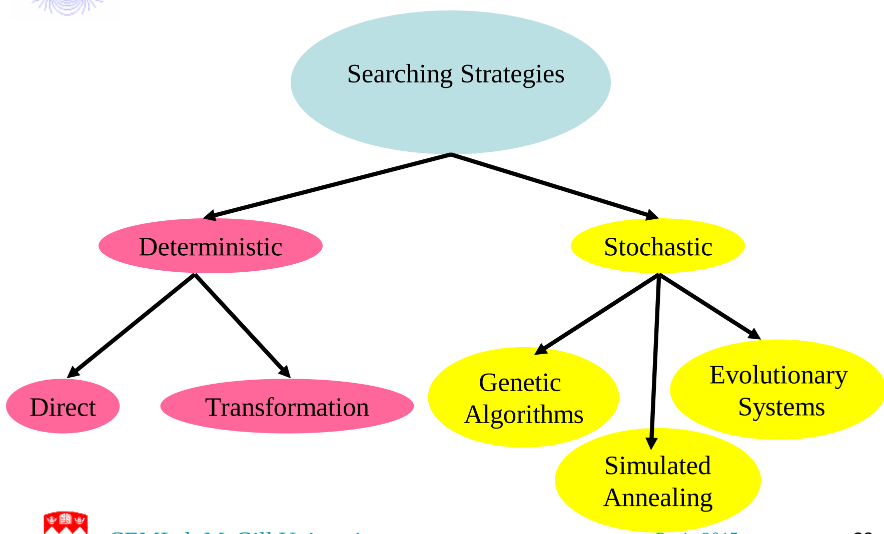
- A design engineer has an appreciation of how a change in a particular parameter will affect the device performance.
  – In other words, he/she has a mental picture of how small changes in any parameter will affect each aspect of the desired performance
  – This is a concept of sensitivity (applied locally)…
- Alternately, if no experience or models exist, random variations can be tried, the performance measured and models developed…

Performing the Search

- The search process can be:
  – Deterministic
    - Needs the local gradient
    - Uses steepest descent
    - Can get stuck in local minima
  – Stochastic
    - Many evaluations of cost function
    - Finds global minimum
    - Often mimics physical or biological systems
      – Evolution
      – Simulated Annealing
      – …



Performing the Search



Optimization

- Ideally, this should be a guided search of the design space within boundaries created by constraints
  - basically consists of a loop consisting of two phases
    - Analysis – estimate performance
    - Modification – change parameters to improve performance

Finding an "Optimal" Design

- A simple process might be described by:
  - Choose a point in the design space (a particular combination of parameters)
  - Evaluate the cost function to locate this device on the hypersurface representing the cost function
  - Determine how to change the parameters to improve the value of the cost function and repeat the previous step
  - Iterate until no further improvement is achieved…

Optimization

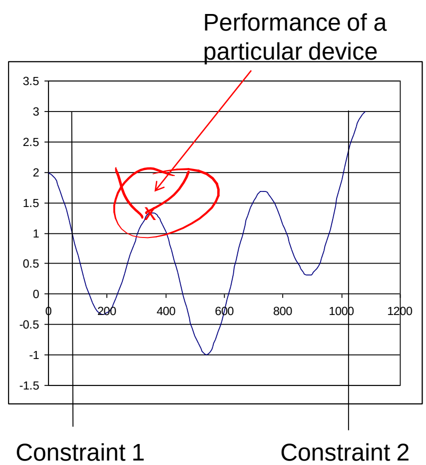
- The key in the process is how the modification phase is implemented..
  - The goal is to minimize a cost function which is constructed from the basic requirements of the device
  - The problem is to determine in which direction to change the device parameters to move towards the minimum

Optimization

- The simplest method of finding the minimum (or maximum) of F(x) is to:
  1. Guess a value for x
  2. Compute F(x)
  3. Compute the gradient of F with respect to x (dF/dx)
  4. Move a small step in the direction of the gradient.
  5. Repeat from 2 until the gradient is zero.
- The problem is finding the derivative – numerical solutions tend to produce the value at a point..

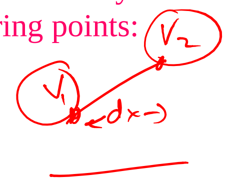
Finding the Derivative

- The problem is that numerical solutions tend to produce the solution at a point..
  - A single “design vector” is provided as input to the analysis
  - The performance values corresponding to the input vector are produced.



Finding the Derivative

- Approach 1: - Finite Difference
  - The gradient of a function can be approximated by evaluating the function at two neighbouring points:



$$F_1 = F(x_1)$$
$$F_2 = F(x_1 + \delta x)$$
$$\frac{dF}{dx} = \frac{F_2 - F_1}{\delta x}$$

  - This requires 2 analysis runs for each gradient evaluation and for each design variable.
  - A lot of work! – expensive…

Finding the Derivative

- Approach 2: - Sensitivity
  - The objective function can be expressed as:

$$F = \int_v f(\mathrm{E}, \mathrm{H})dV$$

  - The fields (E and H) are implicit functions of the systems parameters and the design variables
  - To illustrate the approach, consider an optimization problem which is intended to achieve a specific value of inductance, L_t (assume a linear system).
  - The inductance of the system is given by:

$$L = \frac{2W}{I^2}$$

Sensitivity – Method 1

- If *I* is constant the energy, *W*, is:

$$W = \frac{1}{2\mu_0} A^T S A$$

  - Where S is the coefficient matrix

- So the objective function becomes:

$$f(A,S) = \left(L_T - \frac{1}{I^2 \mu_0} A^T S A \right)^2$$

Sensitivity

- For a design parameter vector, p, the gradient of the objective function is given by:

$$\nabla f = \frac{df}{dp} = \frac{\partial f}{\partial p} + \frac{\partial f}{\partial A}\frac{dA}{dp}$$

$$\frac{\partial f}{\partial p} = -2\left(L_T - \frac{1}{I^2 \mu_0} A^T S A \right) \frac{1}{I^2 \mu_0} A^T \frac{dS}{dp} A$$

$$\frac{\partial f}{\partial A} = -2\left(L_T - \frac{1}{I^2 \mu_0} A^T S A \right) \frac{2}{I^2 \mu_0} S A$$

- To find dA/dp, consider the state equation:

$$SA = C$$

Sensitivity

Then $$\frac{\partial f}{\partial A}\frac{dA}{dp} = \frac{\partial f}{\partial A} S^{-1} S \frac{dA}{dp}$$

Let $$S\frac{dA}{dp} = \frac{dC}{dp} - \left(\frac{dS}{dp}\right) A$$

Now set up the equation $$S^T \lambda = \frac{\partial f}{\partial A}^T = -2\left(L_T - \frac{1}{I^2 \mu_0} A^T S A\right) \frac{2}{I^2 \mu_0} S A$$

This is the system equation where λ is an adjoint variable and $\delta f / \delta A$ represents a set of pseudo sources (in effect magnetic currents)

Sensitivity

$$\nabla f = \frac{df}{dp} = \frac{\partial f}{\partial p} + \lambda^T \left(\frac{dC}{dp} - \left(\frac{dS}{dp}\right) A \right)$$

This equation holds for any term of the objective function that depends on A.

Thus the determination of the gradient requires the solution of the original system plus the adjoint system.

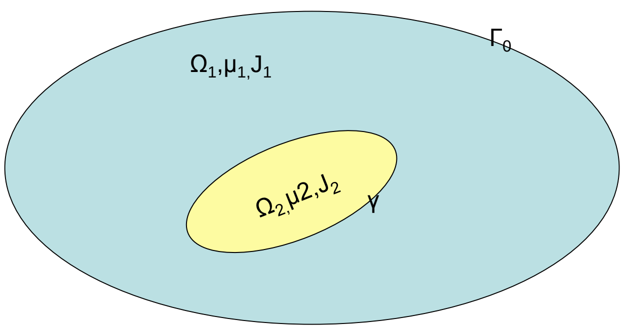
Note that only one adjoint system needs to be solved – it is independent of the number of design variables!

Sensitivity

- This approach has reduced the number of finite element solutions to find the gradient to 2!
- The disadvantage is that the solution of the adjoint problem requires that the sources be set up carefully and that the system matrix is modified.
- This is often referred to as the "Discrete Design Sensitivity Analysis" (DDSA)

Sensitivity – Method 2

Consider the following problem where the shape of region 2 is defined by its boundary, γ, and the objective function includes variations in the material properties, μ, and current densities, J, in the volume and the shape of the region in terms of its boundary.



Sensitivity

– As before, consider an objective function for a magnetostatic system:

$$F = \int_v f(A)dV$$

Where A is a function of the design variable vector, p.

– Note – a full description should include a function on the boundary as well..

$$F = \int_v f(A)dV + \int_\gamma g(A)d\gamma$$

For the following ignore this

– Augment the objective function:

$$F = \int_v f(A)dV + \int_v \lambda\left(-curl\left(\frac{1}{\mu}\right)curlA + J\right)dV$$

Sensitivity

Let

$$\bar{\lambda} = \left(\delta A + \frac{\partial A}{\partial p}\delta p\right), \quad \bar{A} = \left(\delta\lambda + \frac{\partial \lambda}{\partial p}\delta p\right)$$

Then

$$\delta F = \int_v \left[\frac{\partial f}{\partial A}\bar{\lambda}\right]dv - \int_v \left[\frac{1}{\mu}curl\,\bar{\lambda}.curl\lambda + \frac{1}{\mu}curlA.curl\,\bar{A}\right]dv$$

$$+\int_v J.\bar{A}dv + \int_v \left[-\frac{\partial\left(1/\mu\right)}{\partial p}curlA.curl\lambda + \frac{\partial J}{\partial p}.\lambda\right]\delta p dv$$

Sensitivity

Let $\bar{\lambda} = \left(\delta A + \frac{\partial A}{\partial p}\delta p\right), \quad \bar{A} = \left(\delta \lambda + \frac{\partial \lambda}{\partial p}\delta p\right)$

Then

$$\delta F = \int_v \left[\frac{\partial f}{\partial A}\bar{\lambda}\right]dv - \int_v \left[\frac{1}{\mu} curl\,\bar{\lambda}.curl\lambda + \frac{1}{\mu} curlA.curl\,\bar{A}\right]dv$$

$$+\int_v J.\bar{A}\,dv + \int_v \left[-\frac{\partial\left(1/\mu\right)}{\partial p} curlA.curl\lambda + \frac{\partial J}{\partial p}.\lambda\right]\delta p\,dv$$

Variational of original primary system = 0

Sensitivity

$$\delta F = \int_v \left[\frac{\partial f}{\partial A}\bar{\lambda}\right]dv - \int_v \left[\frac{1}{\mu} curl\,\bar{\lambda}.curl\lambda\right]dv + \int_v \left[-\frac{\partial\left(1/\mu\right)}{\partial p} curlA.curl\lambda + \frac{\partial J}{\partial p}.\lambda\right]\delta p\,dv$$

These terms can be set to 0 – they represent the variational of an adjoint system whose "sources" are magnetic currents– the same as in the DDSA system…

So the final sensitivity equation becomes:

$$\frac{dF}{dp} = \int_v \left[-\frac{\partial\left(1/\mu\right)}{\partial p} curlA.curl\lambda + \frac{\partial J}{\partial p}.\lambda\right]dv$$

Note: Because the boundary term was ignored, this equation only considers variations in the properties of volumes..

Sensitivity

- If the boundary integral had not been ignored, the resultant expression would have included the sensitivity to the movement of the boundaries of regions within the problem, i.e. shape sensitivity.
- This approach is known as "Continuum Design Sensitivity Analysis" – CDSA
- Unlike DDSA, it requires no modifications to the matrix setup for the adjoint problem – in fact the method used to solve the field problem is largely irrelevant.

Sensitivity – at last

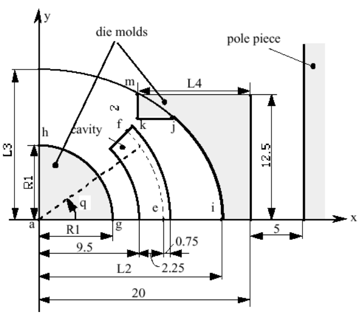
- We now have an approach which can compute objective function sensitivity to all the design variables with one extra solution to the problem…
- In fact, if the objective function is energy based, then the adjoint problem solution is the same as the primary solution and no extra solution is required..
- So, in some cases, the sensitivity (or gradient) of the objective function is free!

Sensitivity – a comment

- The derivation can be done in several ways and it can be shown to be the continuum equivalent to the approach based on Tellegen's theorem in circuit analysis.

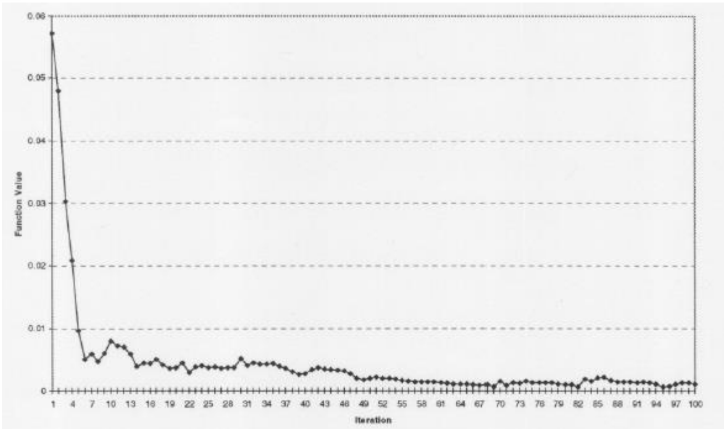
Example

- The Die Press Model (TEAM Problem 25)



$$F = \sum_{i=0}^{8}\left[\left(0.35\cos(2.5+5i) - B_x(11.75, 2.5+5i)\right)^2 + \left[\left(0.35\sin(2.5+5i) - B_y(11.75, 2.5+5i)\right)^2\right]\right]$$

Example



Example

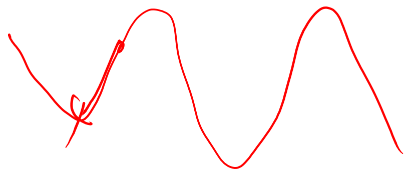
- Results:

	Sensitivity	Brandstatter et al	Takahashi et al	Evolution Strategy
R1	0.00679	0.00705	0.00658	0.00755
L2	0.0133	0.01348	0.01617	0.01571
L3	0.0140	0.01415	0.02617	0.03763
L4	0.0059		0.00502	0.01201

Note – it is not clear that this problem has a single minimum solution..

Issues…

- The gradient driven approach assumes a unique minimum..
  - This is an assumption on the shape of the hypersurface being searched
- What if there are many?



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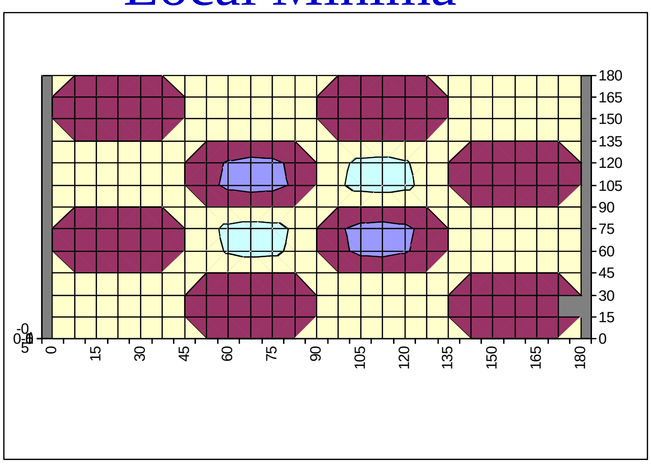
Local Minima



A local minimum may not be the "best" design but the algorithm cannot escape..

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Local Minima

- The problem is that the basic gradient driven approach is really just *Exploitation*
  - It works from an initial guess and will "run downhill"
- To *Explore*, another process is needed.

Searching amongst Multiple Minima

- Use a Stochastic Approach
  - Choose a random starting point and evaluate the cost function.
  - Modify each of the parameters according to the optimizer strategy.
  - Re-evaluate the cost function.
  - If the new device is better, accept it – if it is worse, try an alternate mutation of the starting point.
  - Repeat until all changes result in a worse value of the cost function.

Stochastic Optimization

- There are issues to be addressed:
  - An initial set of "candidates" needs to be chosen to adequately cover the search space
  - After an evaluation of the cost function for all candidates, decisions need to be made on how to modify each candidate to improve them within the local space
  - The new set of candidates needs to be evaluated

This is Exploration

Stochastic Optimization

  - At some point, a set of likely localities for a solution has been identified – these need to be ranked in terms of importance
  - A decision to move to an exploitation algorithm needs to be made – when?
- The answers to these questions result in families of optimization algorithms.

Stochastic Optimization

- Note – stochastic systems rarely use gradient information. Instead they use strategies based on either random variations or the behavior of nearest neighbours.
- Guidelines for possible strategies (algorithms) can come from the natural world:
  - Simulated Annealing
  - Evolutionary Systems
  - …

Conclusions

- Design requires a search of the parameter space
- The search occurs at several levels of abstraction
- Each level provides a virtual environment, with different degrees of approximation, for the simulations

Conclusions

- At each level, the search is an "optimization" process which includes components of "exploration" and "exploitation"
- The goal is to make the overall process as efficient as possible.