# An Introduction to Electrical Machines

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# Contents

**Rotating electrical machines (REM)**
- 1. Ampère law, Faraday principle, Lorentz force
- 2. An overview of REM
- 3. Rotating magnetic field

**Synchronous machine: alternator**

**Solar generator: photovoltaic panel**

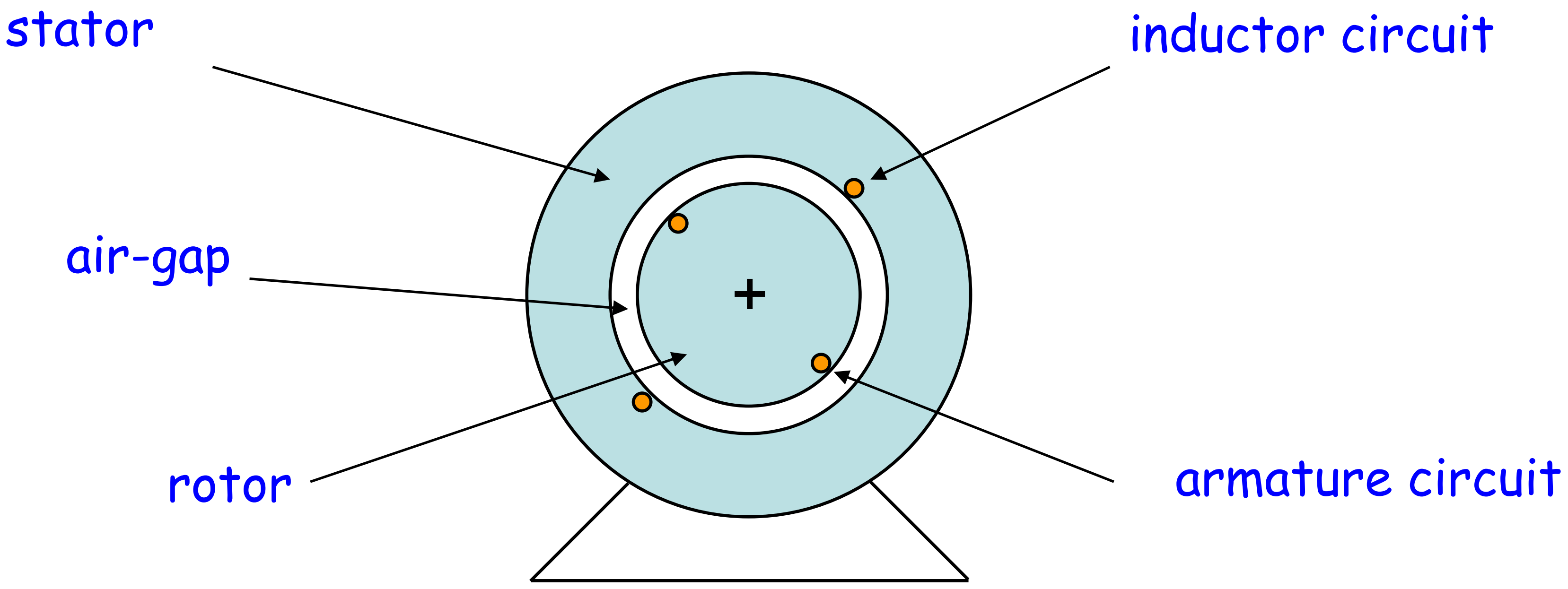
**Asynchronous machine: induction motor**

**Static electrical machines: transformer**
- 1. An overview of the device
- 2. Principle of operation of a single-phase transformer
- 3. Power (three-phase) transformer, signal transformer, autotransformer

## Rotating Electrical Machine

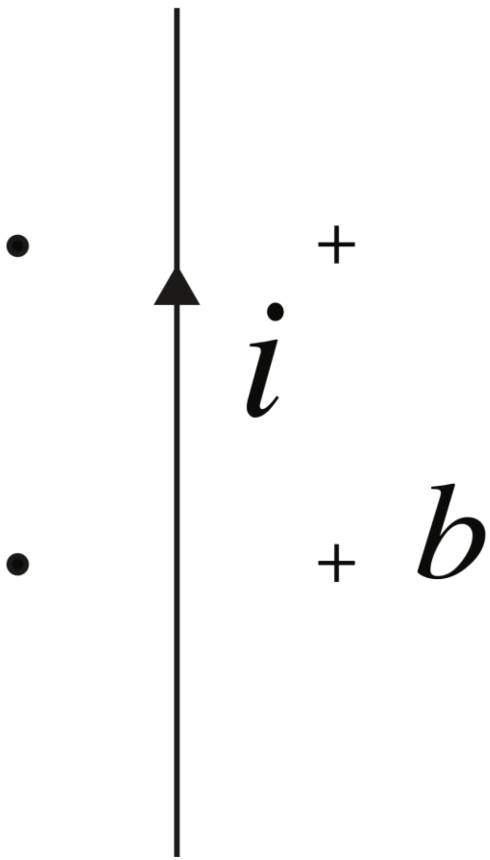
Basically, it consists of:

- an electric circuit (inductor) originating the magnetic field;
- a magnetic circuit concentrating the field lines;
- an electric circuit (armature) experiencing electrodynamical force (motor) or electromotive force (generator).




stator + air-gap + rotor = magnetic circuit

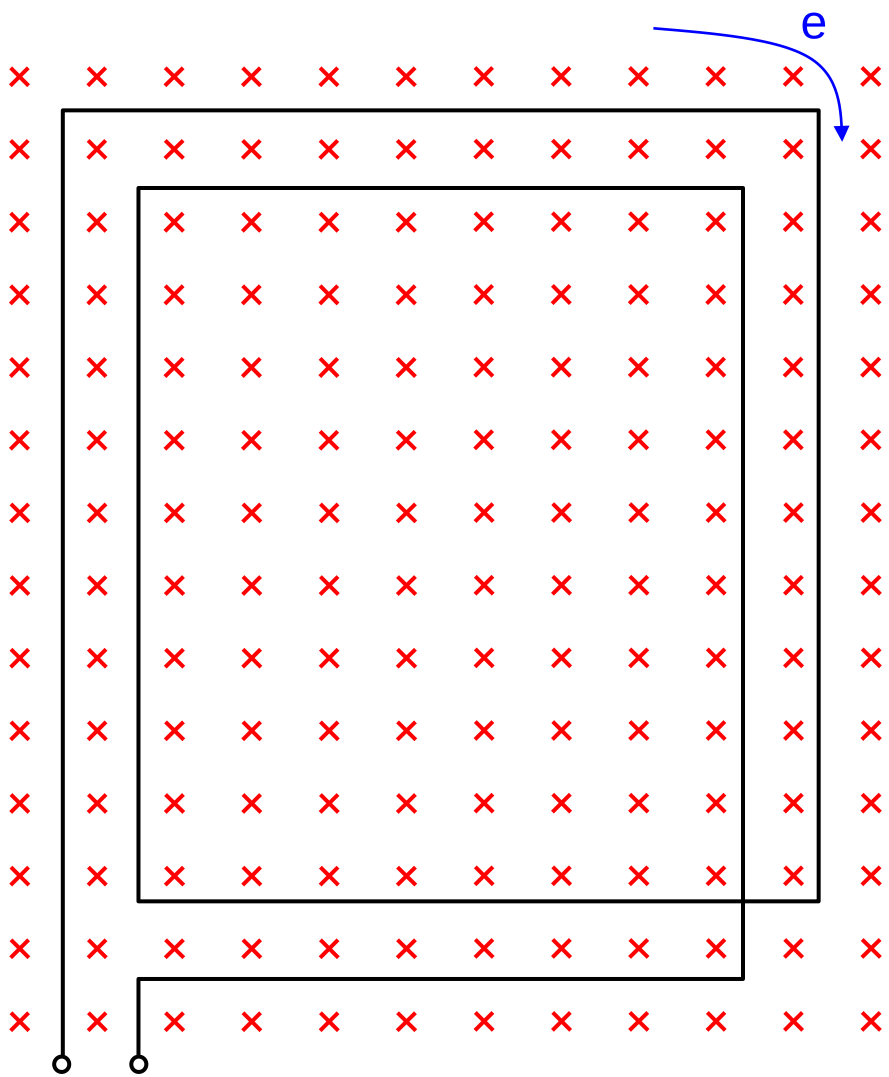
# Principles of magnetics




Ampère law

$$i \rightarrow b$$

# Principles of electromagnetism

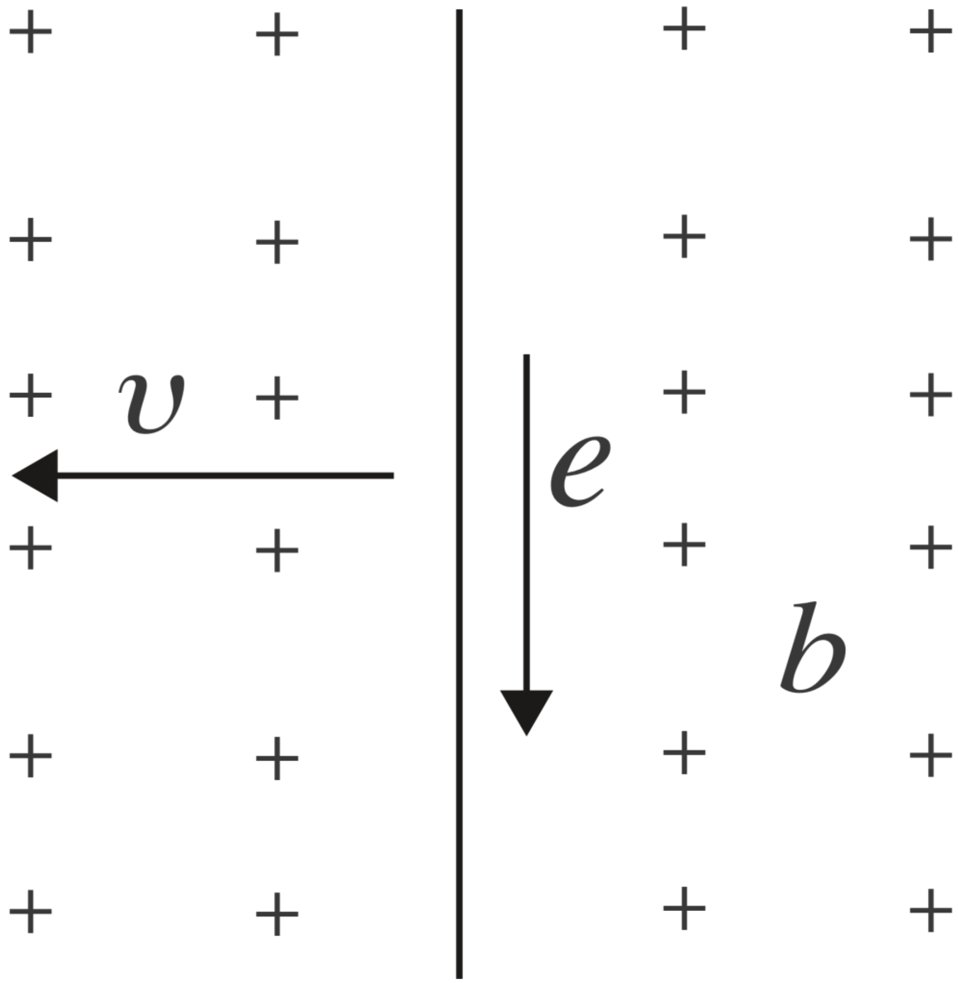



## Faraday-Lenz law

transformer effect

b(t) ⇨ e(t), i(t)

# Principles of electromagnetism

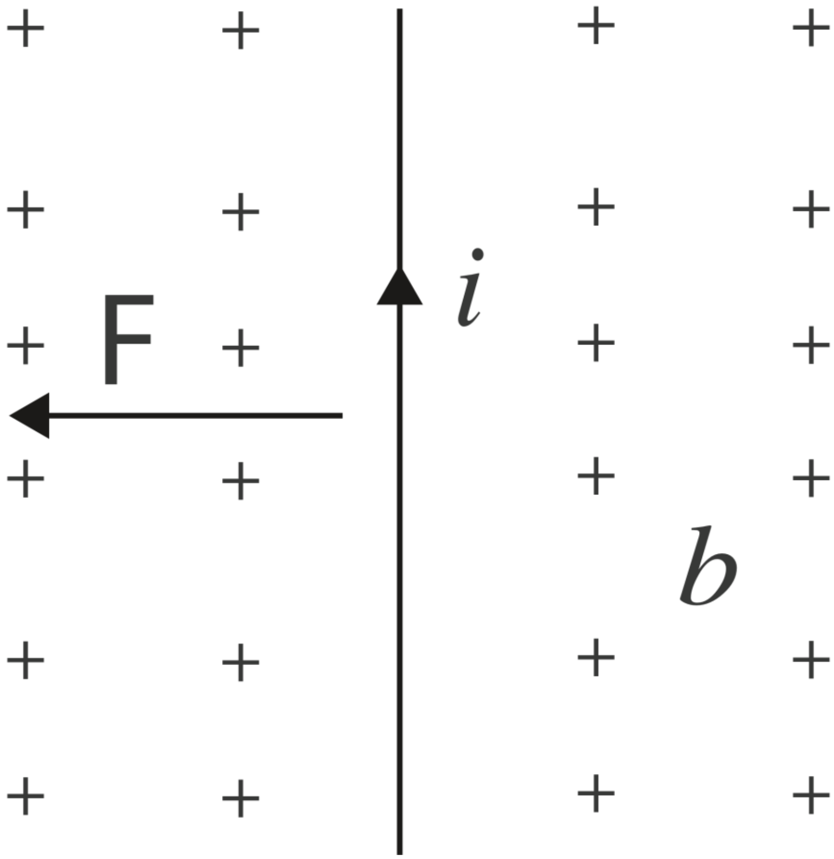



Faraday-Lenz law

(motional effect)

$$\mathrm{b} + \mathrm{v} \rightarrow \mathrm{e}$$

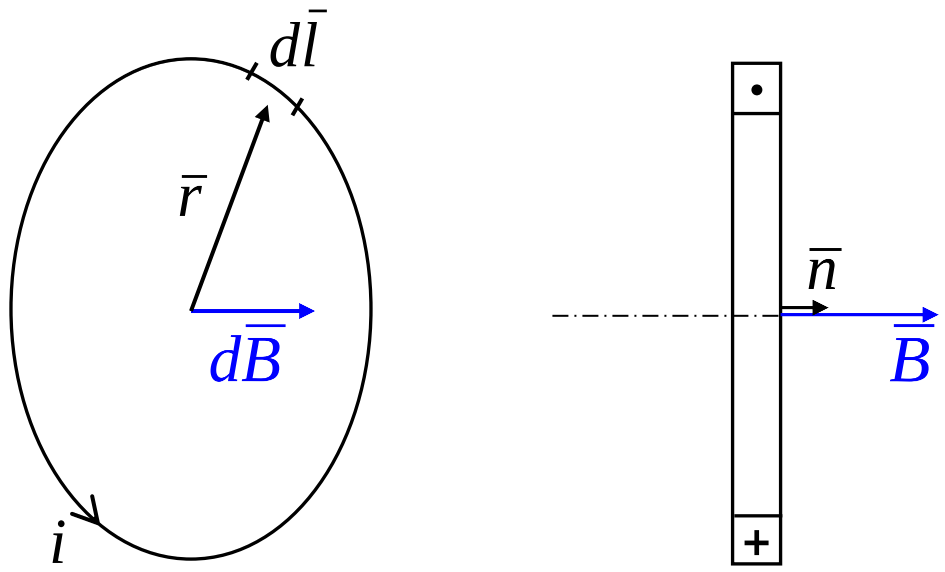
# Principles of electromechanichs




Lorentz force

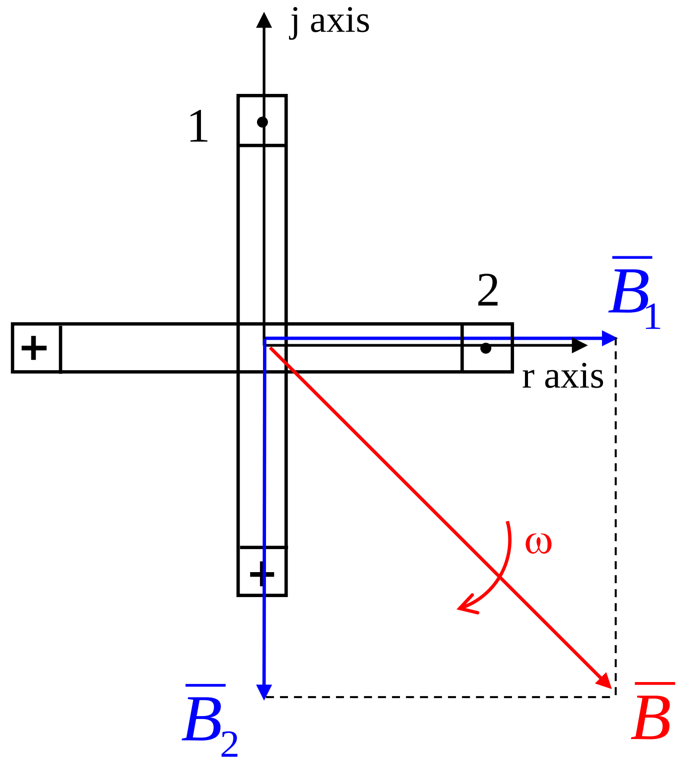
$$b + i \rightarrow F$$

# Magnetic field of a circular coil




$$\bar{B} = \frac{\mu_0 i}{2r}\bar{n}$$

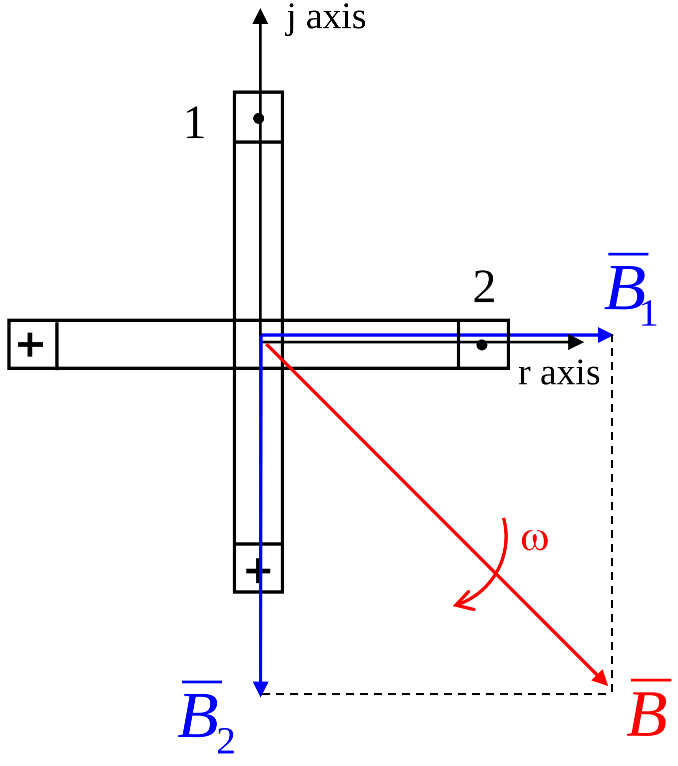
Rotating magnetic field (two-phase) I




$$\overline{B}_1 = \frac{\mu_0 I}{2r}\cos\omega t\,\overline{n}_1 = \frac{\mu_0 I}{2r}\cos\omega t$$

$$\overline{B}_2 = \frac{\mu_0 I}{2r}\cos\left(\omega t - \frac{\pi}{2}\right)\overline{n}_2 = \frac{\mu_0 I}{2r}\sin\omega t\,\overline{n}_2 = -j\frac{\mu_0 I}{2r}\sin\omega t$$

# Rotating magnetic field (two-phase) II



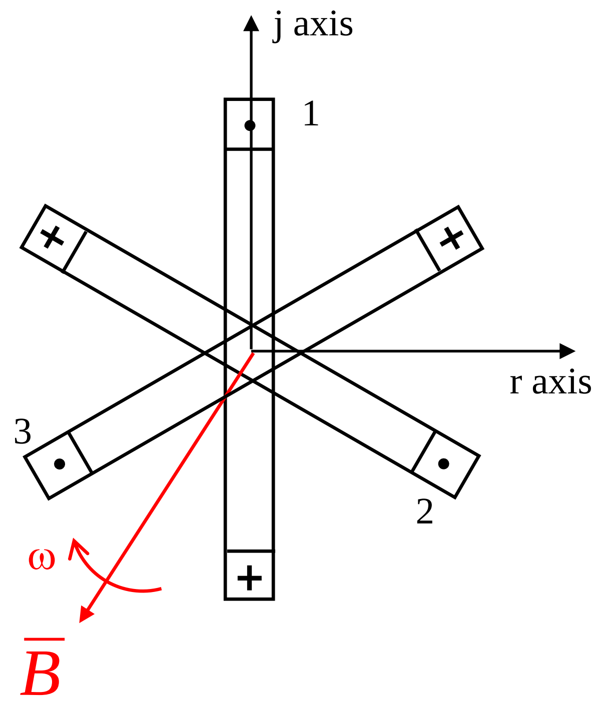

$$\overline{B} = \overline{B}_1 + \overline{B}_2 = \frac{\mu_0 I}{2r}\left(\cos\omega t - j\sin\omega t\right) = \frac{\mu_0 I}{2r} e^{-j\omega t}$$

Rotating magnetic field (three-phase): *Ferraris field*

$$\bar{B}_1 = \frac{\mu_0 I}{2r} \cos \omega t \, \bar{n}_1$$

$$\bar{B}_2 = \frac{\mu_0 I}{2r} \cos\left(\omega t - \frac{2\pi}{3}\right) \bar{n}_2$$

$$\bar{B}_3 = \frac{\mu_0 I}{2r} \cos\left(\omega t - \frac{4\pi}{3}\right) \bar{n}_3$$




$$\bar{B} = \frac{3}{2} \frac{\mu_0 I}{2r} e^{-j\omega t}$$

Two-pole rotating field
Field speed = angle frequency $\omega = 2\pi f$ [rad $s^{-1}$] = 60f [rpm]

# Field speed and pole number

Given the angle frequency $\omega = 2\pi f$ [rad $s^{-1}$]

it turns out to be that p (number of poles) and $N_0$ (field speed) are inversely proportional:

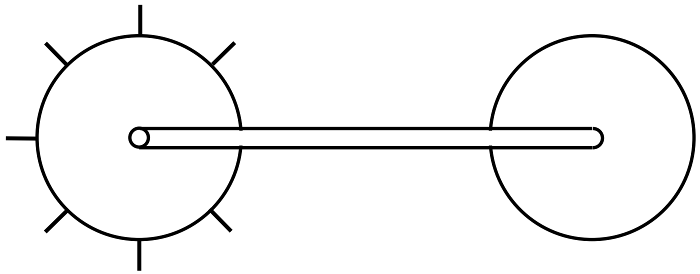
$$N_0 = 120\, f / p \quad \text{[rpm]}$$

Rationale

360° ➔ one mechanical cycle

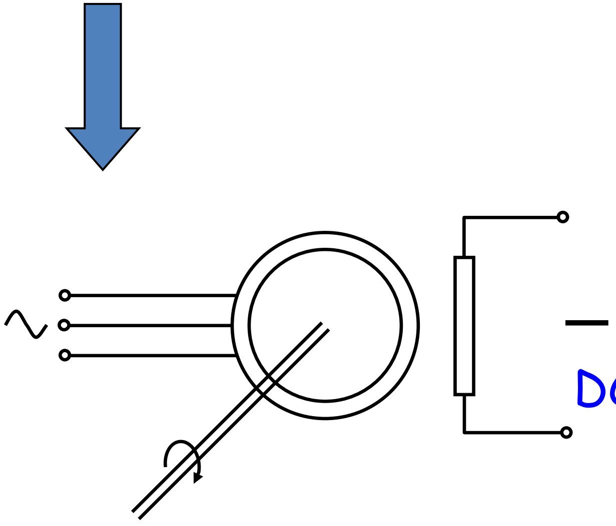
360° : p/2 ➔ one magnetic cycle

# Synchronous generator



turbine
(hydraulic or vapour)

alternator



Three-phase AC output

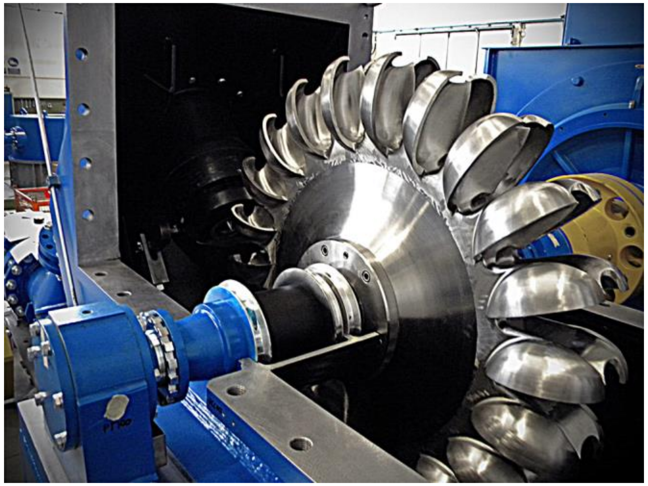
DC excitation

Mechanical input

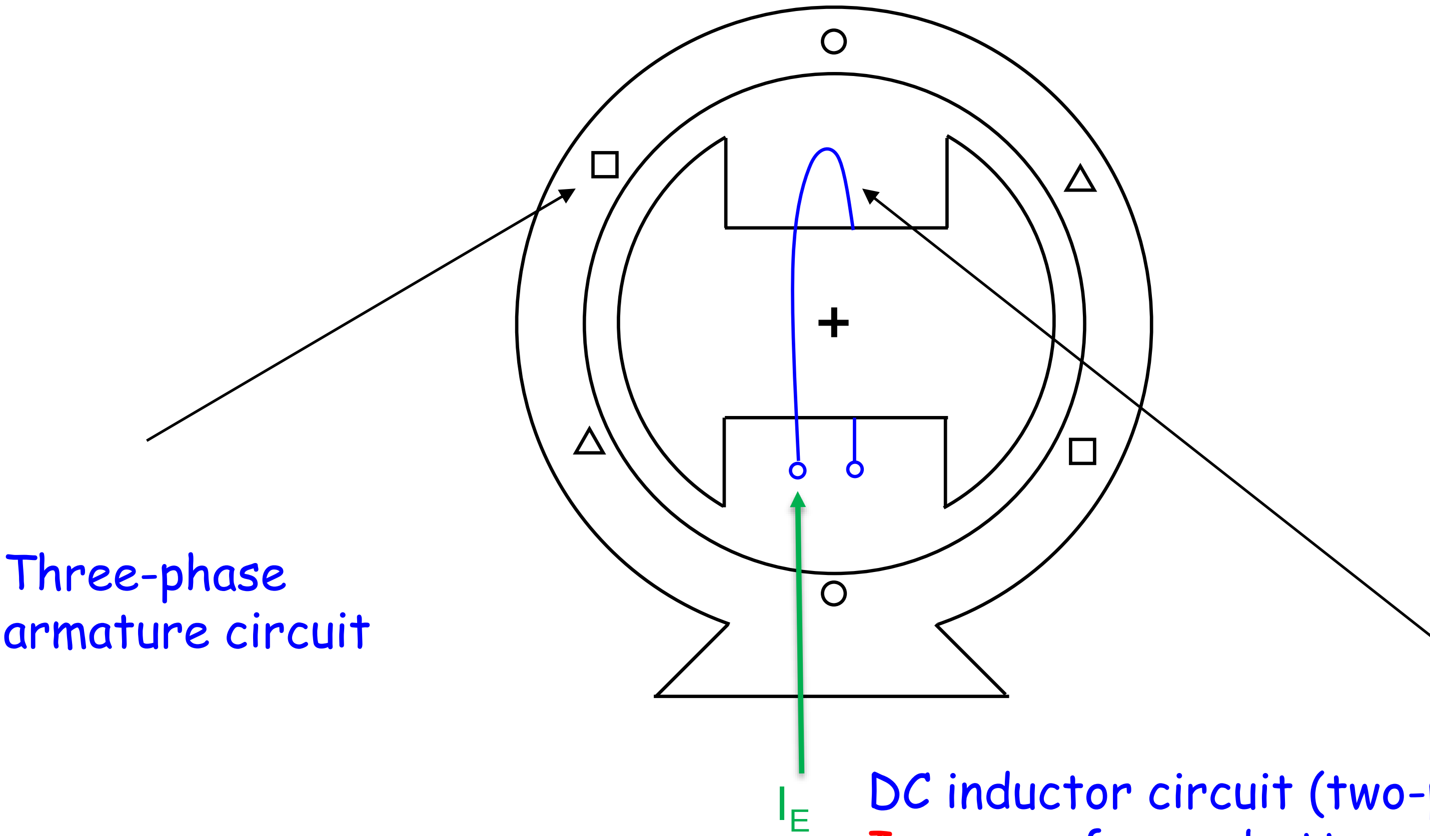
# Hydraulic turbines

Kaplan turbine
small jumps 10 m
large flows 200 $m^3/s$

Pelton turbine
large jumps 1000 m
small flows 20 $m^3/s$

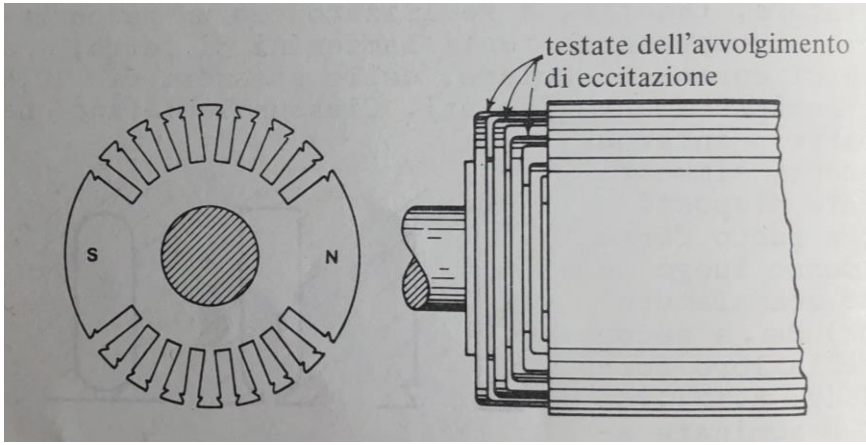


# Alternator structure




DC inductor circuit (two-pole)
$I_E$ comes from a battery or a dynamo
In low-power alternators the circuit can be replaced by a permanent magnet

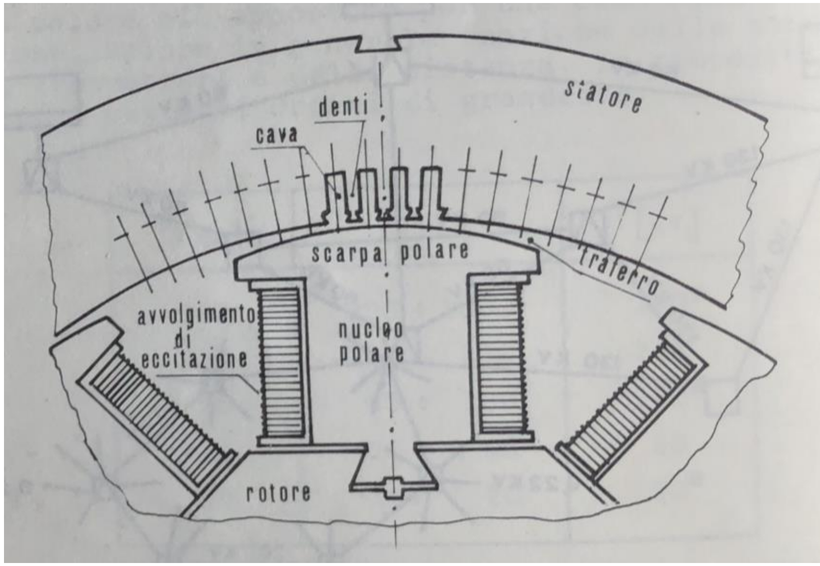
As a generator, it converts mechanical power into electric power.




<u>Smooth pole rotor</u>

2 poles, turboalternator
vapour turbine

Rotor material: laminated iron
Shaft material: non-magnetic steel




<u>Salient pole rotor</u>

4 – 6 – 8- 10 – 12… poles
hydraulic turbine

Core material: laminated iron
Rotor material: non-magnetic steel

## Alternator operating principle

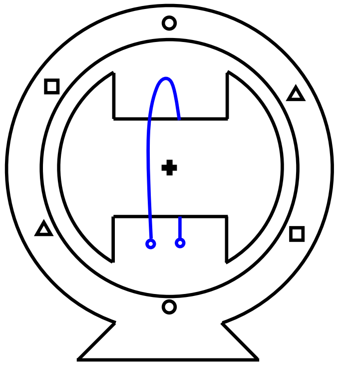
- The inductor circuit is supplied by a DC current $I_E$
- A magnetic flux $\Phi_R$ sinusoidally distributed in the air-gap is originated
- A speed $\Omega$ is applied to the rotor shaft
- In the armature circuit a three-phase symmetric electromotive force

  $E = k\Omega\Phi_R$ at an angular frequency $\omega = \Omega p/2$ is induced
- If the armature circuit is on load, a three-phase (balanced) current I is delivered, so providing an active power $P = 3EI \cos \phi$ (resistive load)
- An equal amount of mechanical power must be supplied to the shaft
- Current I generates a synchronous rotating field (speed $\omega$), which originates a flux $\Phi_S$ opposing flux $\Phi_R$
- In order to keep E constant, excitation current $I_E$ must be varied

## Alternator operating principle: a closer insight

- In un conduttore, posto in una cava dell'indotto di lunghezza $\ell$, in moto relativo a velocità v ortogonalmente alle linee di flusso di B, si induce una f.e.m. di valore efficace

$$E = \frac{B_M \ell v}{\sqrt{2}}$$



Indicando con:

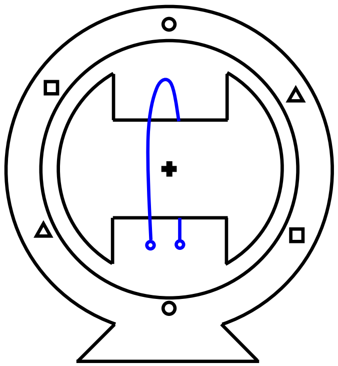
- $\Phi$ flusso utile per polo (flusso entrante nell'indotto attraversando un semipasso polare $\tau$)
- f, frequenza di variazione dell'induzione nell'indotto al ruotare del rotore
- $k_f$ fattore di forma (rapporto tra valore efficace e valor medio in un semiperiodo)

la f.e.m. è esprimibile come

$$E = 2 k_f \Phi f$$

- Ogni conduttore di indotto è soggetto alla stessa f.e.m. efficace

## Alternator operating principle: a closer insight



Infatti

$$\Phi = B_m \tau \ell = \frac{2}{\pi} B_M \tau \ell \quad \Rightarrow \quad B_M = \frac{\pi \Phi}{2\tau\ell}$$

Inoltre $v = 2\tau f$ essendo v T = 2 τ

e, in regime P.A.S. $k_f = \frac{\pi}{2\sqrt{2}} = 1.11$

- Legame tra frequenza f, numero di giri N (rpm), numero di poli p:

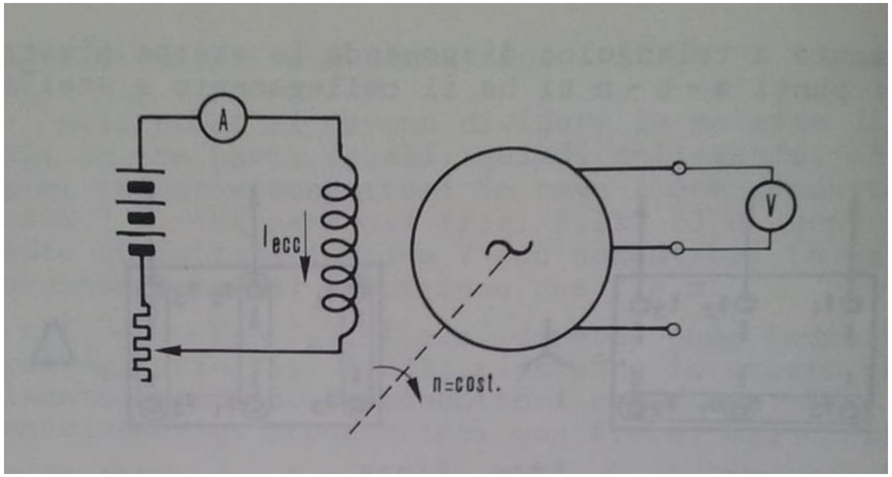
$$f = \frac{pN}{120}$$

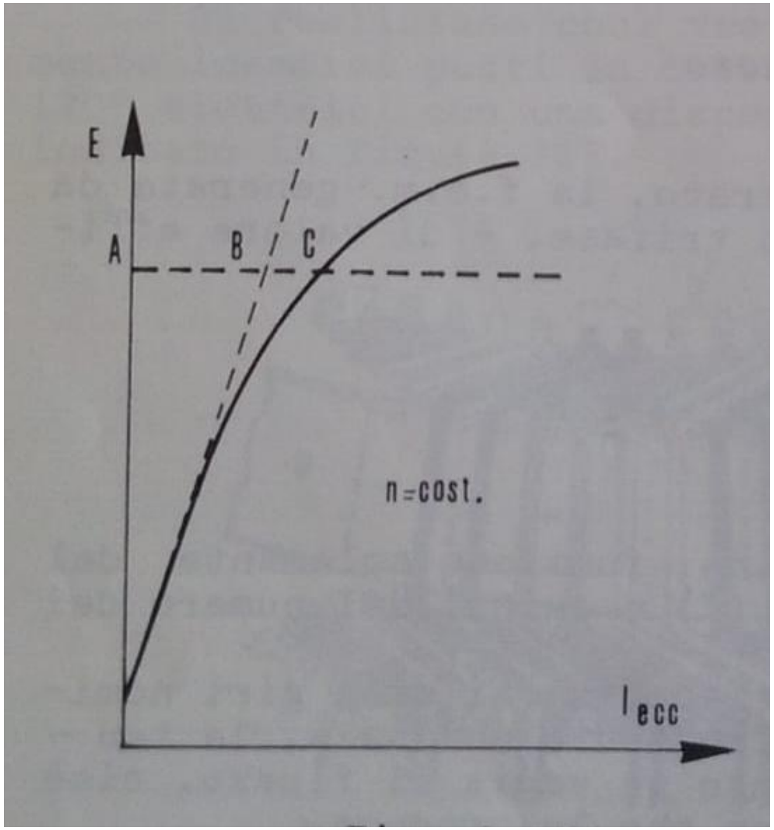
- La frequenza della tensione generata è una grandezza unificata (50 Hz in Europa, 60 Hz negli USA)

- Fissato il numero di poli, la velocità dell'alternatore è dettata dalla rete elettrica cui è asservito

p = 2 – 4 – 6 – 8 – 10 – 12 …

# Funzionamento a vuoto dell'alternatore




Ai morsetti di una fase:

$E = 2k_f \, \Phi \, f \, p \, n \, q \, k_e$

n, conduttori per cava
q, cave per polo e per fase
p, poli
$k_e$ , coefficiente di tensione (<1)
$N_t$ = p n q, numero totale dei conduttori serie per fase

# Funzionamento a carico dell'alternatore

## Reazione di indotto

Carico resistivo: correnti in fase

Carico induttivo: correnti in quadratura in ritardo

Carico capacitivo: correnti in quadratura in anticipo

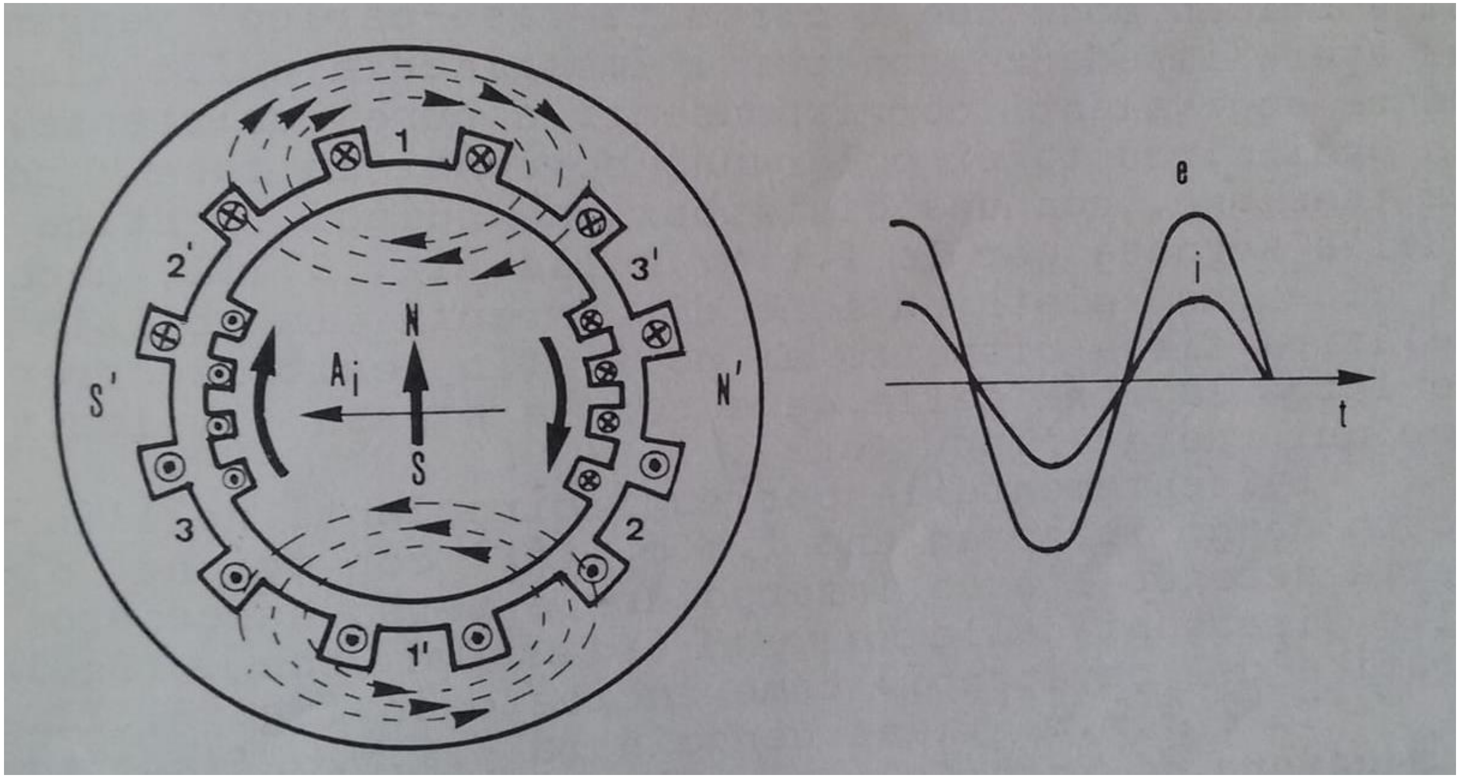
Reazione: forza magnetomotrice di indotto (valore max)

$M_i = 1.31\ n\ q\ I$

n, conduttori per cava
q, cave per polo e per fase
I, corrente efficace

## Reazione di indotto (1)

Carico resistivo: correnti in fase
- forze tangenziali: coppia frenante
- effetto smagnetizzante: $E < E_0$

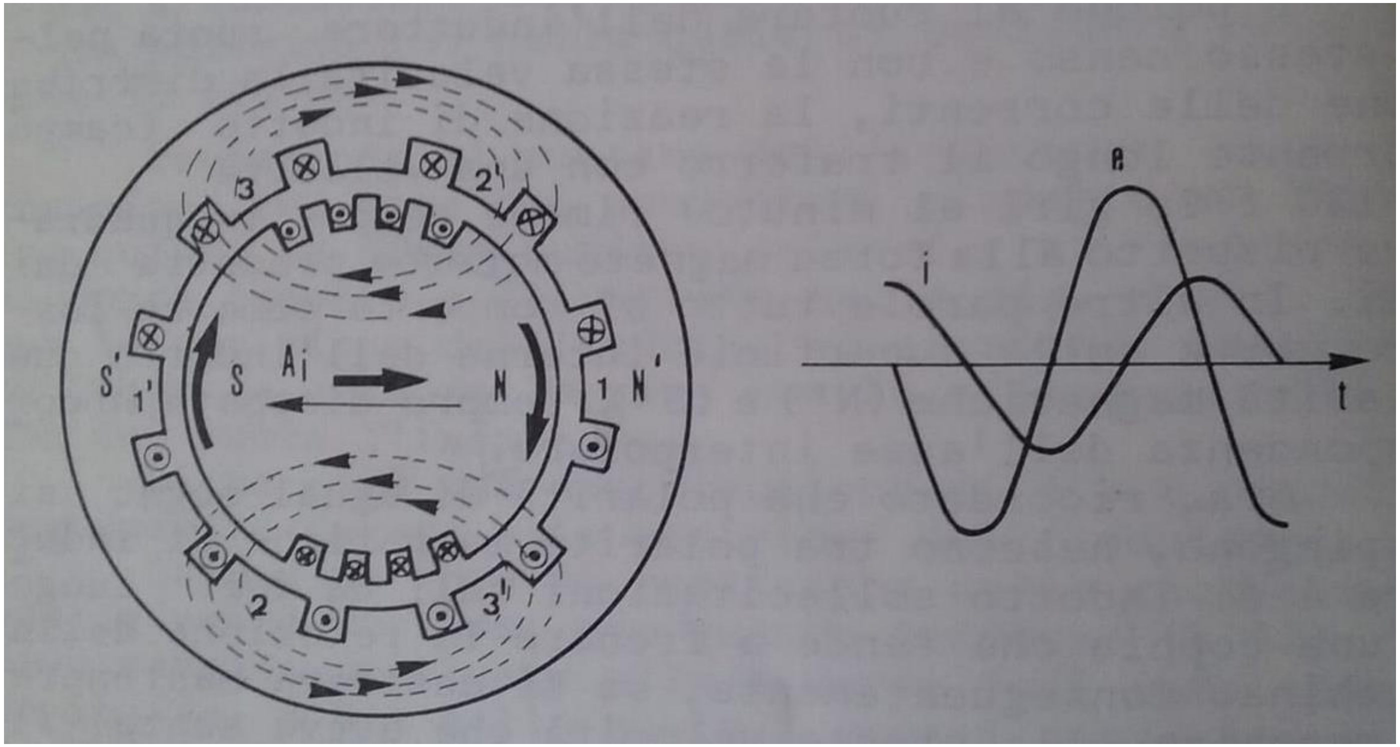



2 cave per polo per fase = 12 cave

## Reazione di indotto (2)

Carico induttivo: correnti in quadratura in ritardo
- forze radiali: conduttori in compressione
- effetto smagnetizzante $E < E_0$

Equivale a un *condensatore trifase rotante*

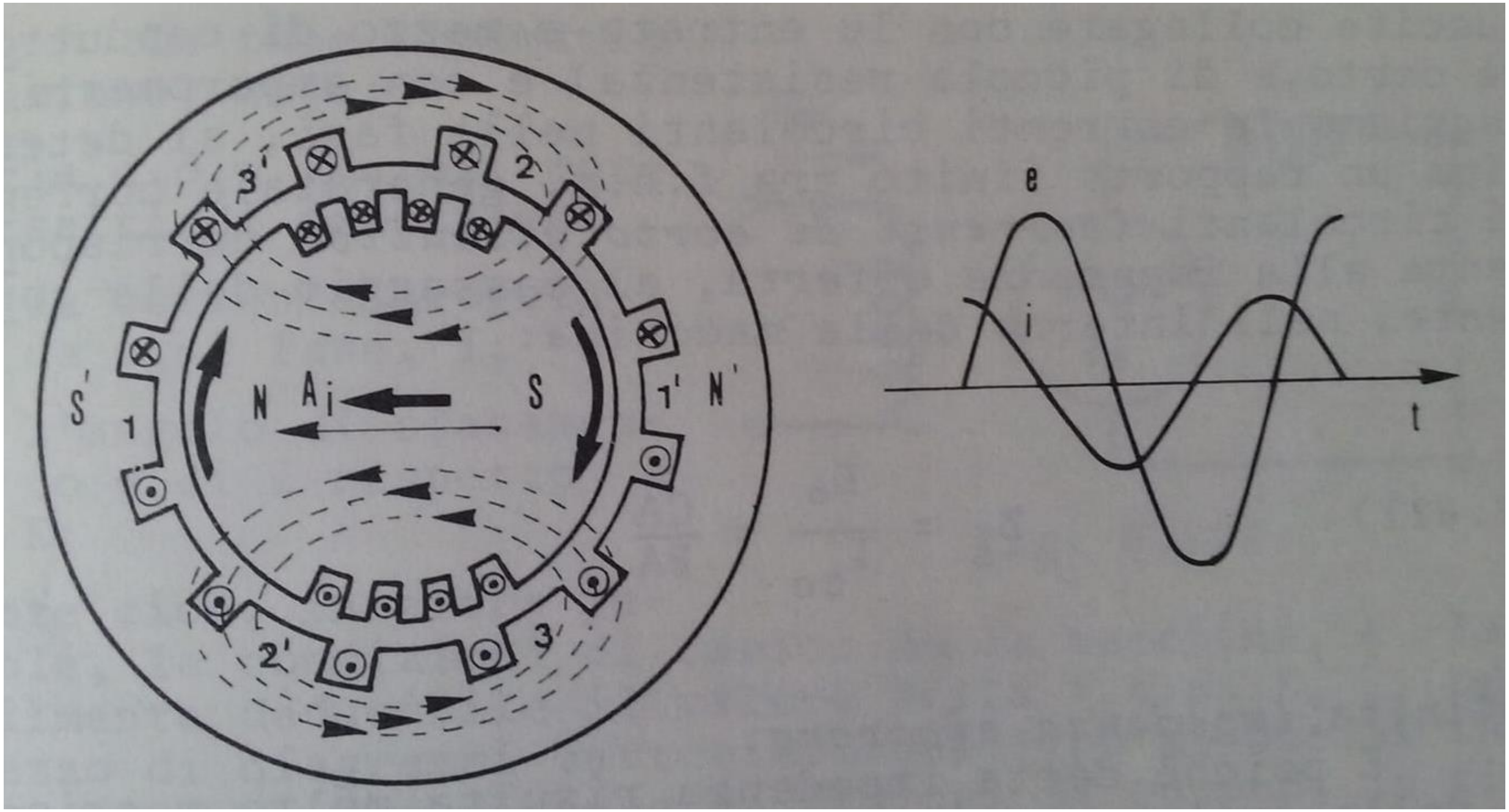


## Reazione di indotto (3)

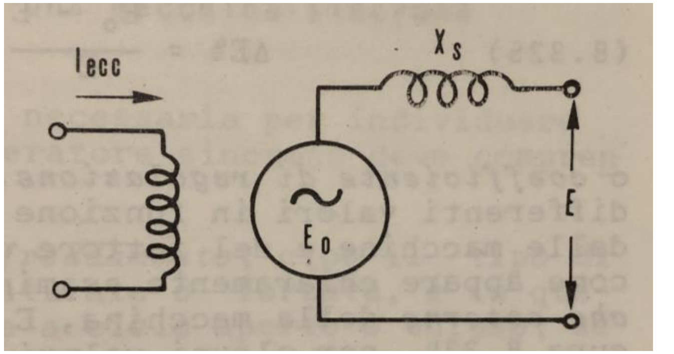
Carico capacitivo: correnti in quadratura in anticipo

- forze radiali: conduttori in trazione
- effetto magnetizzante: $E > E_0$

Equivale a un *induttore trifase rotante*



## Regolazione della corrente di eccitazione $I_E$




Impedenza sincrona di fase :

$Z_s = E_0/I_{CC} = R_s + jX_s$ (Thévenin/Norton ai morsetti di una fase)

KVL alla maglia del carico, che assorbe corrente I sotto la tensione E :

$E_0 \sim E \cos\phi + j\, E \sin\phi + j\, X_s I$ KVL

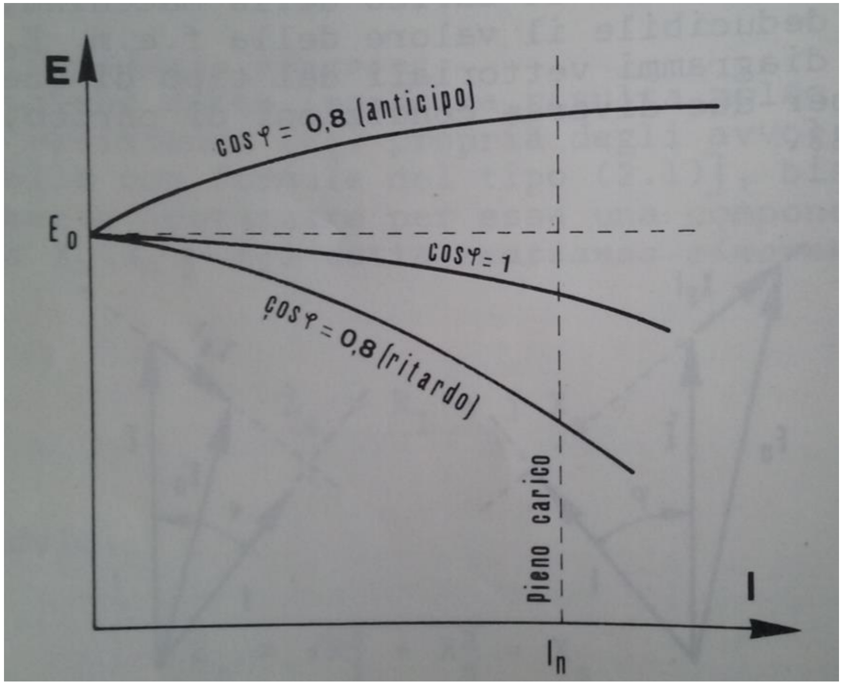
$E_0 = [\, (E \cos\phi)^2 + (E \sin\phi + X_s I)^2 \,]^{1/2}$

A propria volta $E_0$ dipende dalla corrente di eccitazione $I_E$ :

$E_0(I_E) = 2k_f\Phi(I_E)f$

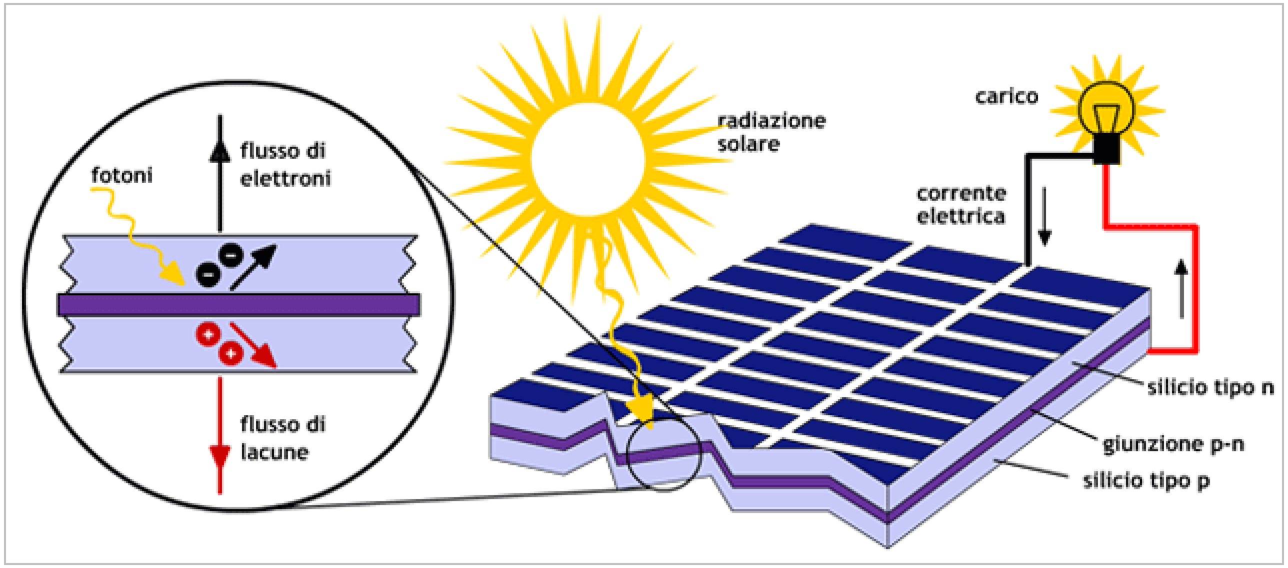
Si corregge $I_E$ e quindi $\Phi$ e quindi $E_0$ in maniera tale da

- erogare $EI\cos\phi$ (watt 'attivi') al carico
- controllare $\Delta E/E$ al variare del carico

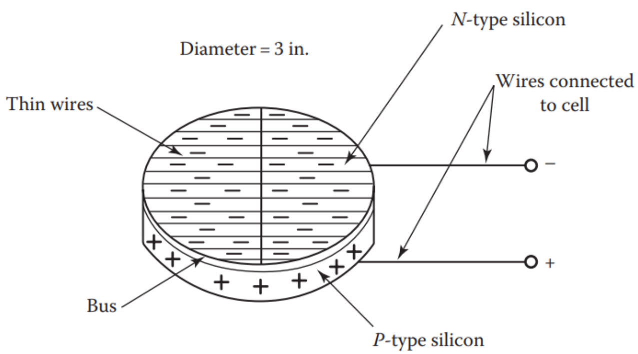


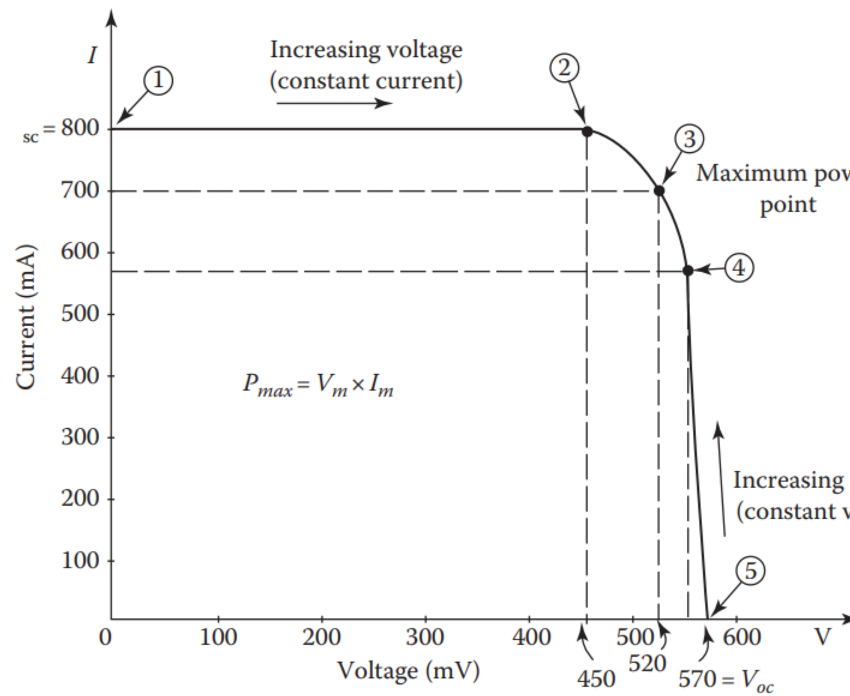

giri ed eccitazione costanti

# Generatori solari: cella fotovoltaica

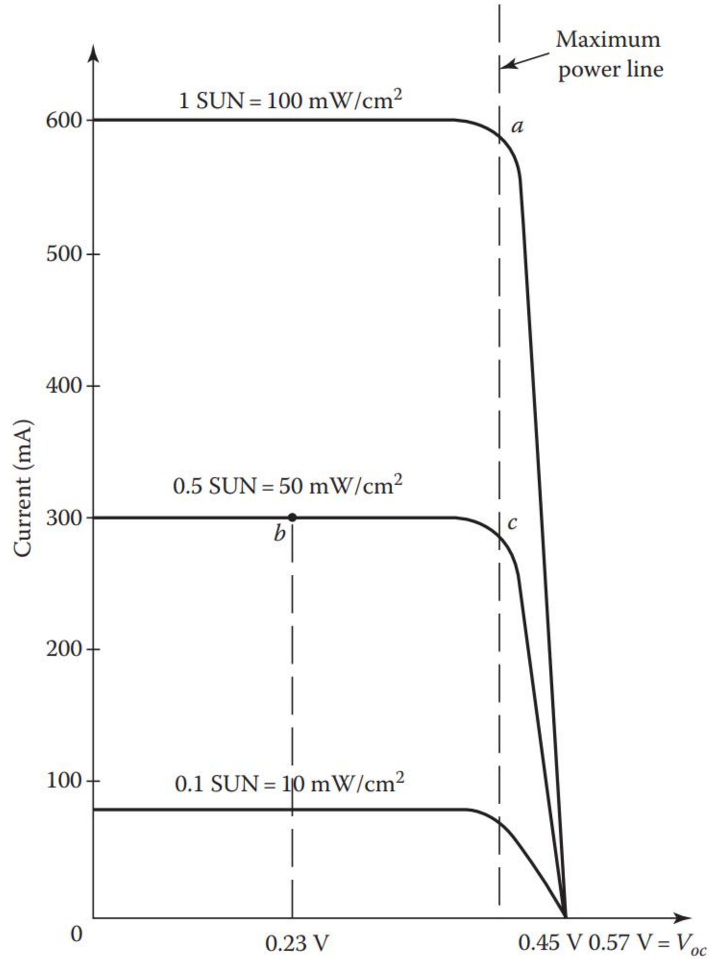


# Solar energy systems - 1



# Solar energy systems - 2



# Temperature effect

$$E_0 = E_R - 0.0021(T - 25) \qquad (13.3)$$

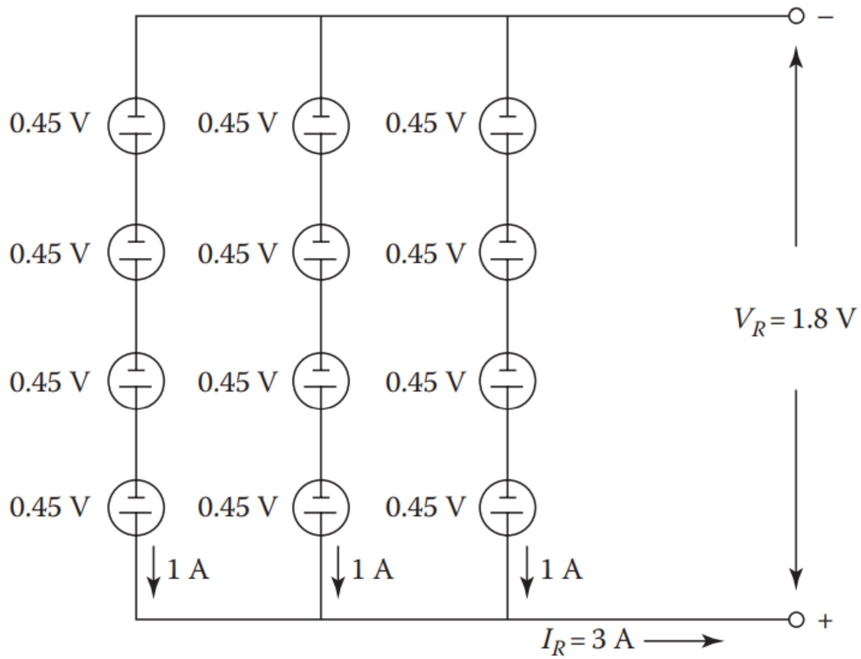
and

$$I_0 = I_R + 0.025A(T - 25) \qquad (13.4)$$

where $E_R$ and $I_R$ are the cell ratings in volts and mA at 25°C. $E_0$ and $I_0$ will be the cell voltage and current at the new temperature $T$ in degrees Celsius. $A$ is the cell area in square centimeter.

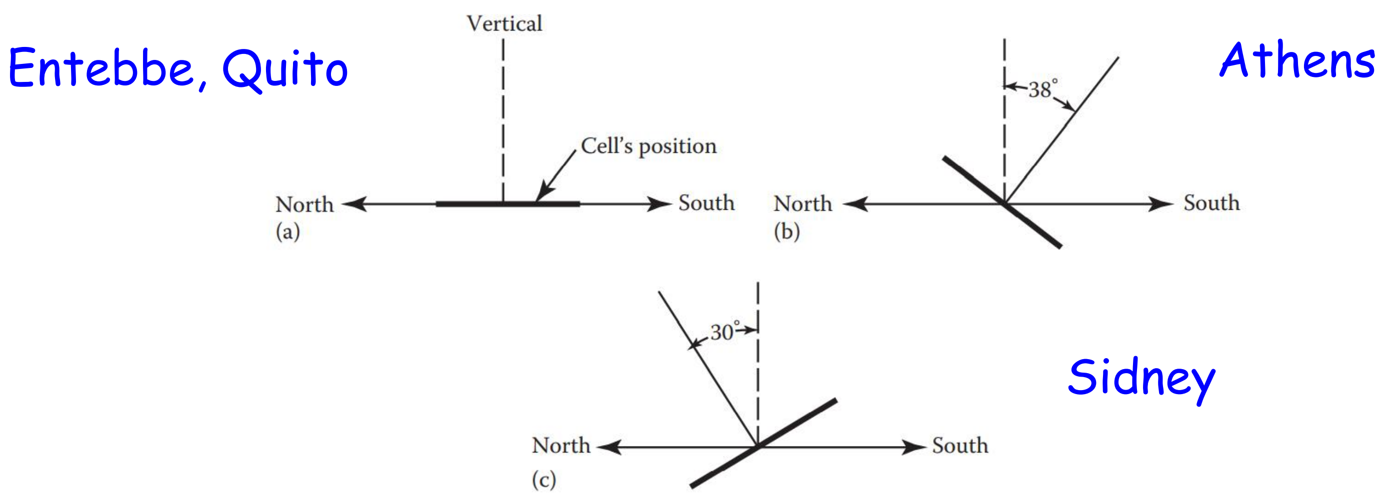
Cells operate more efficiently when they are cooler

# From solar cell to solar panel

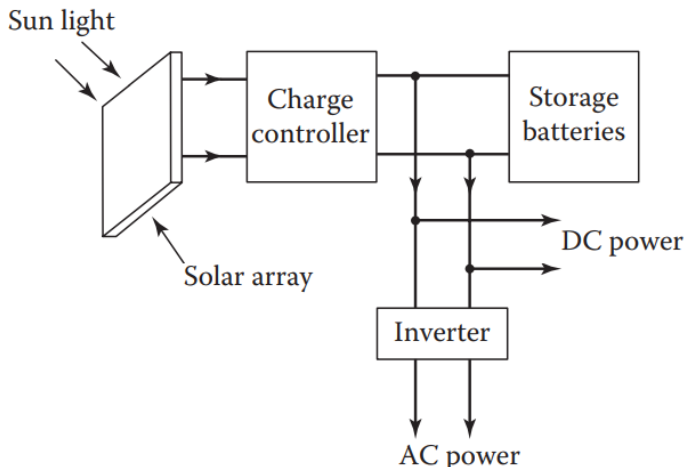


## The Latitudes of Selected Cities around the World

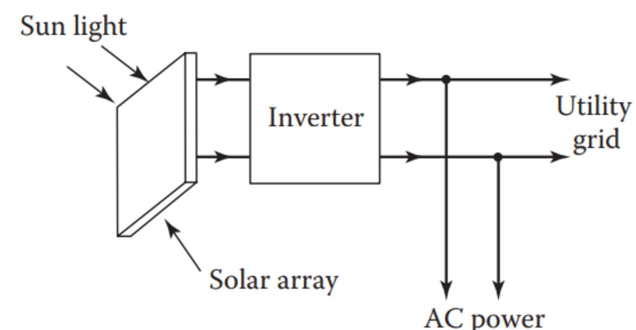
| Location | Latitude | Location | Latitude |
|---|---|---|---|
| Athens, Greece | 38° N | Madrid, Spain | 40° N |
| Berlin, Germany | 53° N | Miami, Florida | 26° N |
| Bogota, Columbia | 2° N | Montreal, Quebec | 46° N |
| Bombay, India | 20° N | Moscow, Russia | 55° N |
| Buenos Aries, Argentina | 20° N | Munich, Germany | 48° N |
| Cairo, Egypt | 30° N | Oslo, Norway | 60° N |
| Edinburgh, Scotland | 56° N | Paris, France | 49° N |
| Entebbe, Uganda | 0° | Quito, Ecuador | 0° |
| Honolulu, Hawaii | 20° N | Rio de Janeiro, Brazil | 23° S |
| Houston, Texas | 30° N | Rome, Italy | 42° N |
| Kansas City, Missouri | 39° N | Seattle, Washington | 47° N |
| Las Vegas, Nevada | 36° N | Sidney, Australia | 35° S |
| Lima, Peru | 12° S | Thule, Greenland | 77° N |
| London, England | 52° N | Tokyo, Japan | 36° N |
| Los Angeles, California | 34° N | Valparaiso, Chile | 36° N |



# Solar generator configurations




(a)

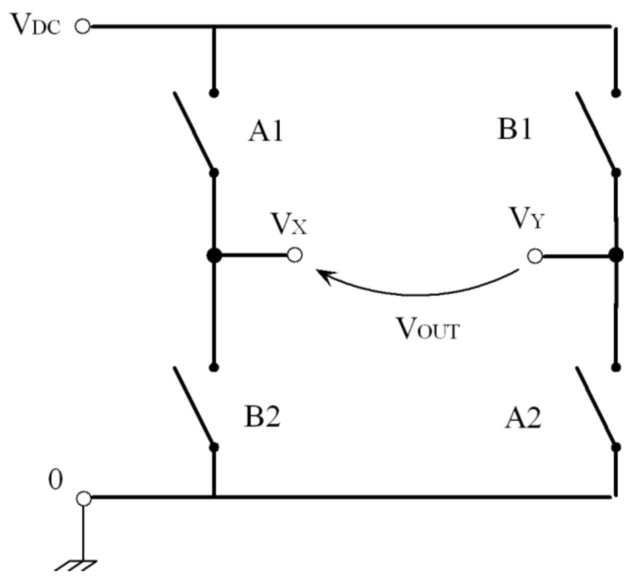


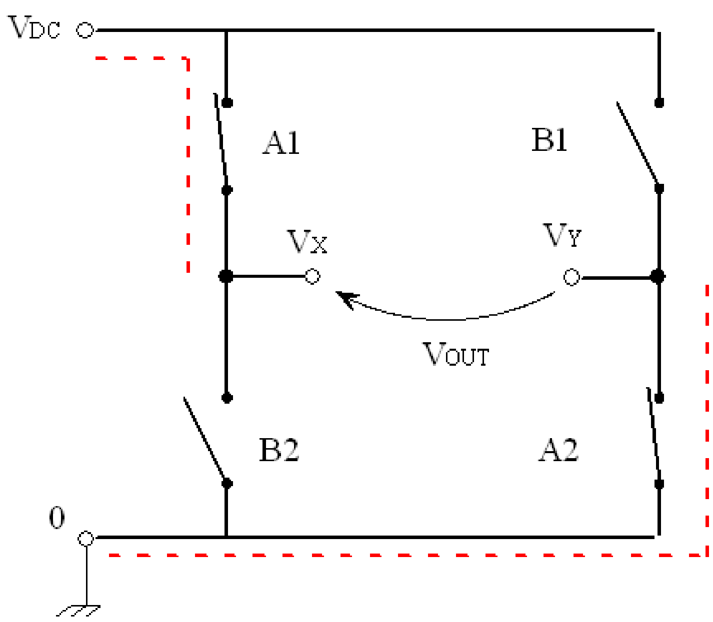

(b)

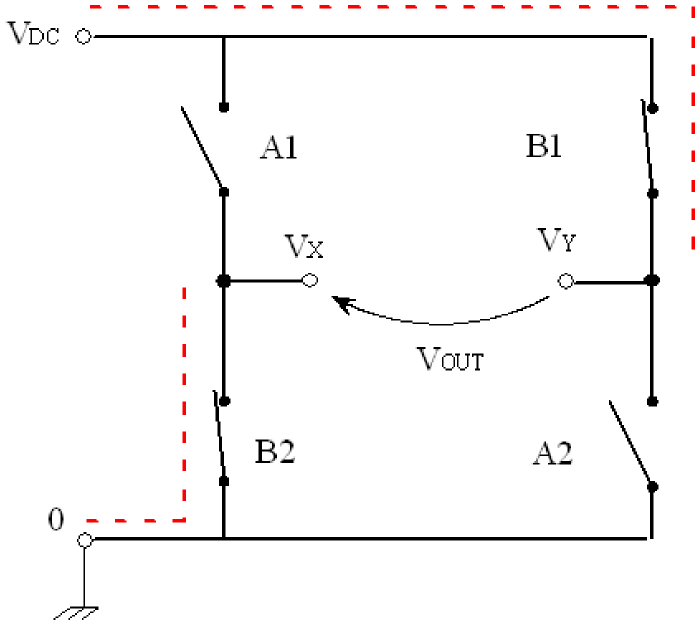
Stand-alone system

Co-generation system

# Inverter operation principle



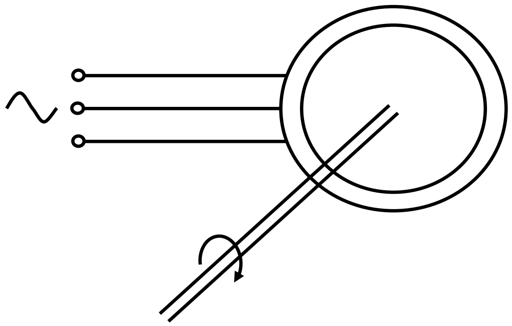



# Three-phase asynchronous motor (induction motor)

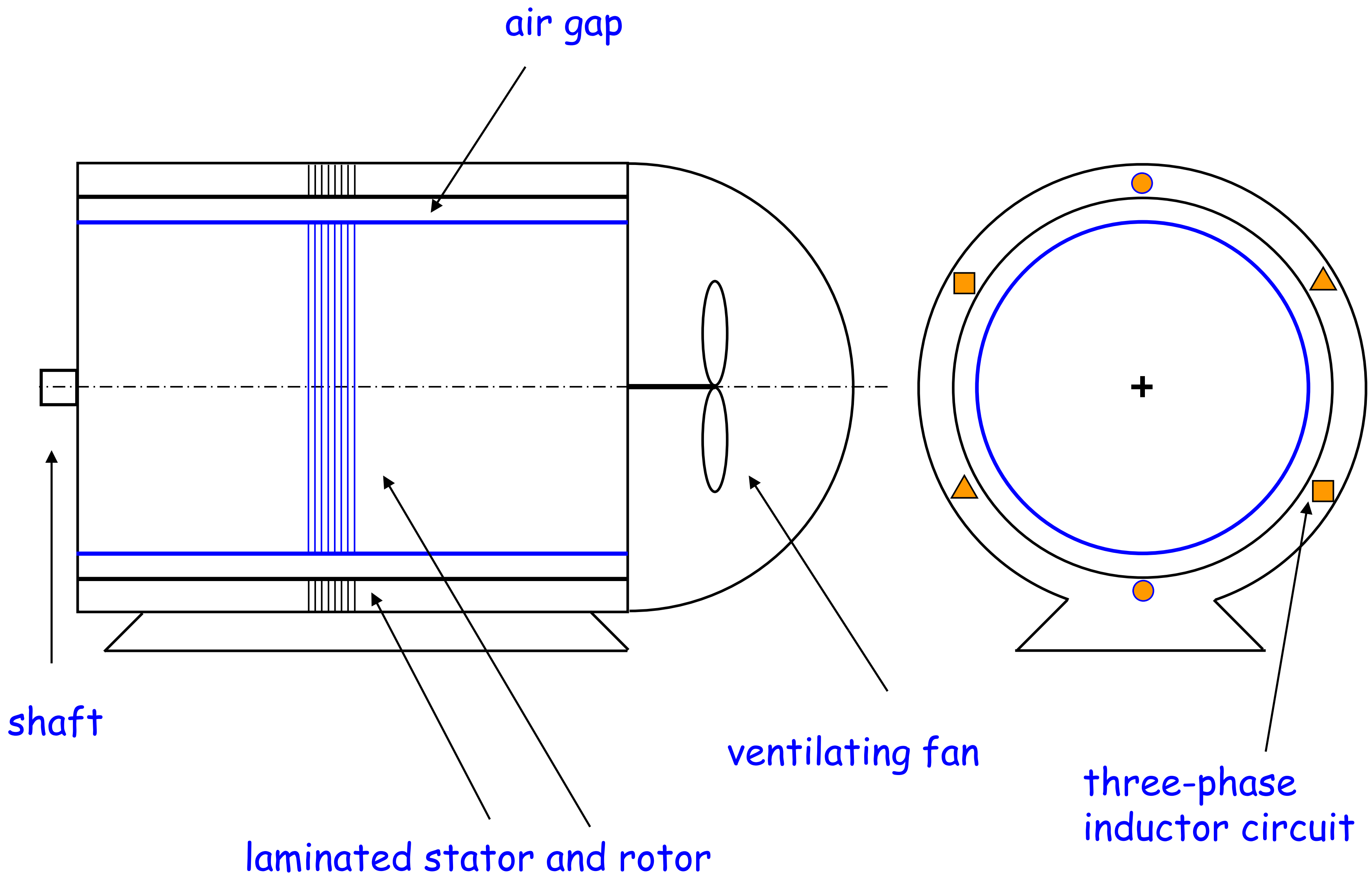
Graphic symbol

Three-phase AC input

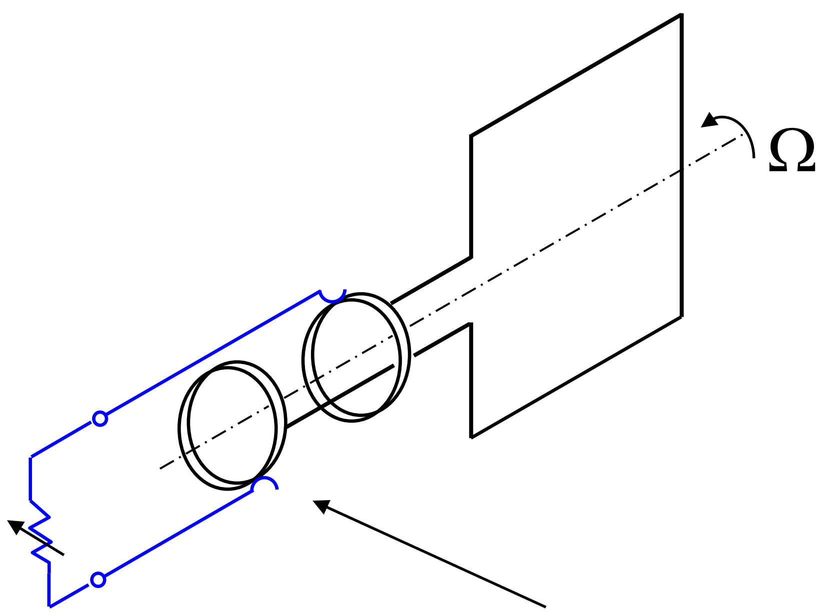


Mechanical output

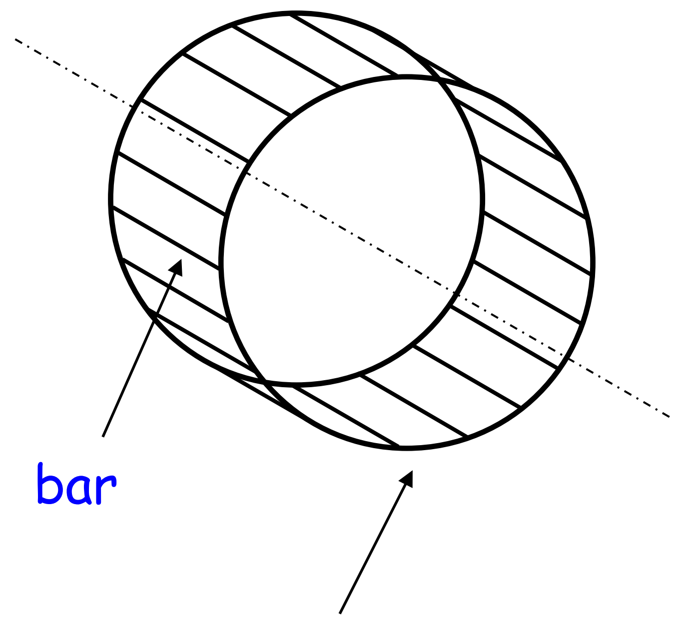
As a motor, it converts electric power into mechanical power


air gap
shaft
laminated stator and rotor
ventilating fan
three-phase
inductor circuit

## Wound rotor




## Squirrel-cage rotor



## Induction motor: operating principle

- Inductor terminals are supplied by a three-phase symmetric voltage $V_1$
- A three-phase balanced current $I_1$ is delivered to inductor windings.
- An induction field $B_1$ rotating at speed $N_0 = 120\ f\ p^{-1}$ is originated (f current frequency, p number of poles, speed in rpm).
- A sinusoidal magnetic flux $\Phi$ takes place in the magnetic circuit (stator-air gap-rotor)
- In stator and rotor windings, subject to sinusoidal magnetic flux $\Phi$,

electromotive forces

$E_1=2.22\ m_1\ f\ \Phi$ and $E_2=2.22\ m_2\ f\ \Phi$

are induced, respectively, such that $E_1/E_2 = m_1/m_2 = a$

$\Phi$ max value of flux, $E_{1,2}$ RMS-value of electromotive force
$m_{1,2}$ number of conductors in a winding

$E_2$ ➡ three-phase balanced current $I_2$ in the rotor windings

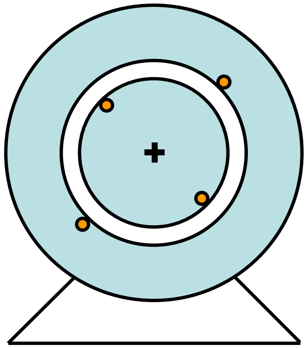
Magnetic effect of $I_2$ :

- a synchronous rotating field $B_2$ (speed $sN_0$) is originated, which tends to decrease the flux inductor flux $\Phi$ of rotating field $B_1$

- flux $\Phi$ and so $B_1$ depend on the applied voltage $V_1$ and cannot vary

- voltage source $V_1$ delivers an extra current $I_3$ to the inductor, which originates a mmf that compensates the mmf of rotor current $I_2$

Mechanical effect of $I_2$ :



the axially-directed conductors

placed in the radially-directed field $B_1$

experience tangentially-directed forces $F_t$

The rotor experiences a torque $C=2RF_t$ : if unconstrained, it rotates.

Having defined the slip factor $s=(N_0-N)/N_0$
as the relative speed of the field with respect to the rotor, it turns out to be:

- the frequency of $E_2$ and $I_2$ is $f_2=sf$
- the rotor speed is $N=(1-s)N_0$
- the speed of the field wrt the rotor (slip speed) is $sN_0$
- stator and rotor fields rotate in the air gap at the same synchronous speed $N_0$

If the braking torque is equal to zero
then, the running torque $C_M=0$ and therefore

| | | |
|---|---|---|
| $f \neq 0$ | $I_2=0$ | $E_2=0$ |
| $f_2=0$ | $s=0$ | $N=N_0$ |

the rotor is synchronous wrt the rotating field

If the braking torque is different from zero
then, the running torque $C_M \neq 0$ equals the braking torque and therefore

| | | |
|---|---|---|
| $f \neq 0$ | $I_2 \neq 0$ | $E_2 \neq 0$ |
| $f_2 \neq 0$ | $0 < s <= 1$ | $0 <= N < N_0$ |

the rotor is asynchronous wrt the rotating field
(the rotor follows the field)

## Torque-speed curve

Torque C is proportional to both flux $\Phi_1$ of the rotating field and real part of the armature current $I_2 \cos \phi_2$ (active component):

$$C = k\,\Phi_1 I_2 \cos\varphi_2$$

Current $I_2$ and power factor $\cos \phi_2$ depend on emf $E_2$ and armature circuit impedance:

$$Z_2 = \sqrt{R_2^2 + (s\omega L_2)^2}$$

resistive-inductive impedance $R_2 + js\omega L_2$

$$I_2 = \frac{E_2}{Z_2} \quad , \quad \cos\varphi_2 = \frac{R_2}{Z_2}$$

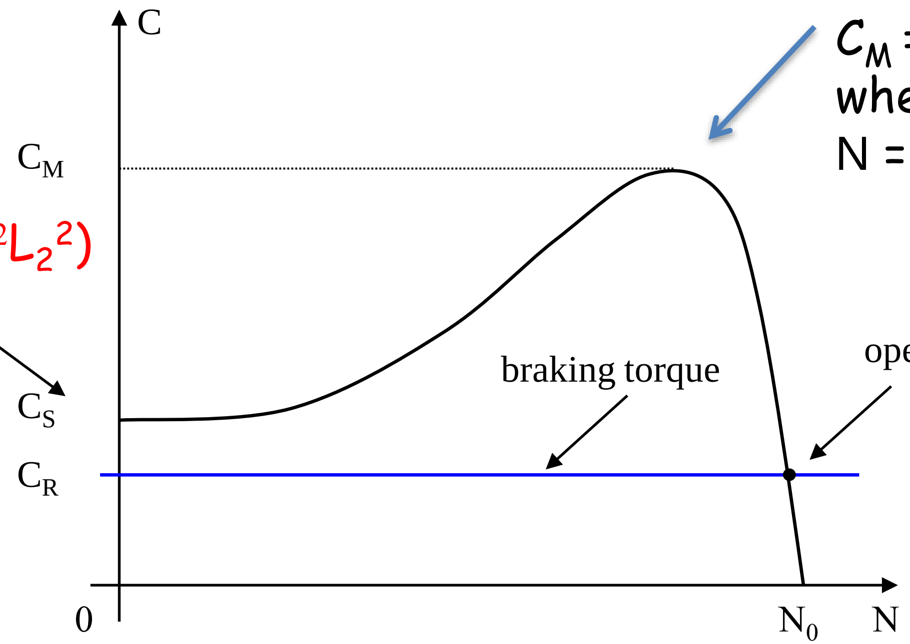
On the other hand, electromotive force $E_2$ depends on both relative rotor speed $N_0 - N$ and flux $\Phi_1$

$$E_2 = k'(N_0 - N)\Phi_1$$

It turns out to be:

$$C = k\,\Phi_1 \frac{E_2}{Z_2}\frac{R_2}{Z_2} = k_1 \Phi_1^2 \frac{N_0^2 (N_0 - N) R_2}{(N_0 R_2)^2 + (N_0 - N)^2 (\omega L_2)^2} \Rightarrow C = C(N)\ ,\ C \propto \Phi_1^2 \propto V_1^2$$

## Torque-speed curve




Max torque
$C_M = k_1\Phi_1^2 N_0/(2\omega L_2)$
when
$N = N_0(1-R_2/\omega L_2)$

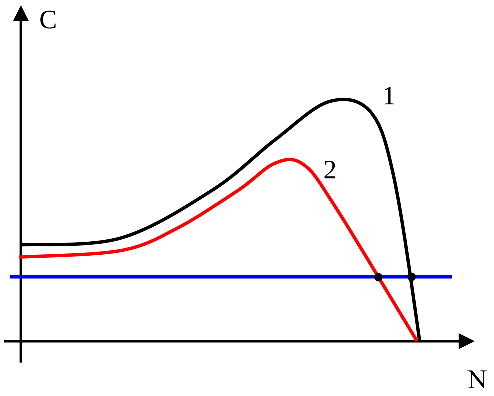
self-starting
$C_s = k_1\Phi_1^2 N_0 R_2/(R_2^2+\omega^2 L_2^2)$

synchronous speed (rpm) $$N_0 = \frac{120f}{p}$$

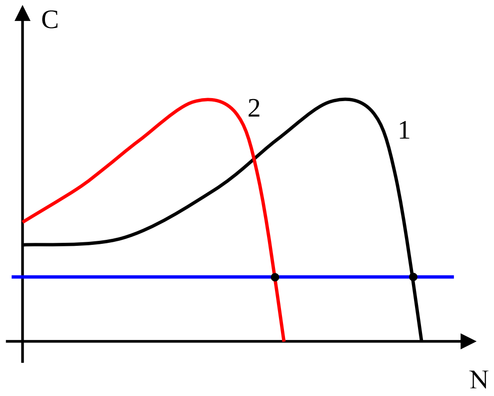
mechanical power $P_m = CN$ (output, at the shaft)
electric power $P_e = \sqrt{3}\, VI \cos\phi$ (input, at the terminals)
Loss less (ideal) motor : $P_e = P_m$ (power balance)

# Speed control

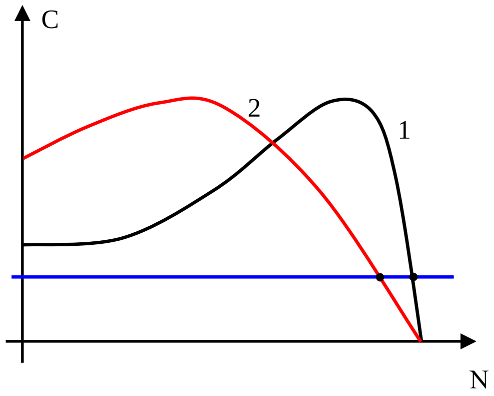
decreasing V




decreasing f




increasing $R_2$
(wound rotor only)




In the past: also pole commutation technique

Esempio

Un motore asincrono trifase di potenza 20 kW, alimentato con tensione concatenata di 380 V a 50 Hz, nel funzionamento a pieno carico ha rendimento 0,91, fattore di potenza 0.9 e scorrimento 3,3 % . L'induttore è formato da avvolgimenti a stella che a propria volta formano tre coppie polari, mentre il rotore è a gabbia.

Allo spunto a pieno carico, con l'alimentazione di 380 V, la corrente assorbita dal motore sia 5,8 volte quella del funzionamento normale.

Trovare:
- la corrente assorbita I e la coppia motrice C nel funzionamento a pieno carico;
- la corrente $I_a$ e la potenza nominale $A_n$ all'avviamento con alimentazione di 380 V assumendo una coppia di spunto uguale alla coppia motrice a pieno carico

| | |
|---|---|
| potenza elettrica | $P = P_m / r = 20 \text{ kW} / 0{,}91 = 22 \text{ kW}$ |
| corrente a pieno carico | $I = P / [\ \text{sqrt}(3)\ V \cos\phi\ ] = 37 \text{ A}$ |
| velocità di sincronismo | $N_0 = 120\ f / p = 120 \times 50 / 6 = 1000 \text{ rpm}$ |
| velocità rotore | $N = N_0 (1 - s) = 1000 \times (1 - 0{,}033) = 967 \text{ rpm} = 100 \text{ rad/s}$ |
| coppia motrice | $C = P_m / N = 20 \text{ kW} / 100 = 200 \text{ Nm}$ |
| corrente all'avviamento | $I_a = 5{,}8 \times 37 = 214{,}6 \text{ A}$ |
| potenza all'avviamento | $A_n = \text{sqrt}(3) \times 380 \times 214{,}6 = 141{,}2 \text{ kVA}$ |

Esempio (cont.)

Inoltre trovare:
- la tensione necessaria V' per lo spunto con coppia resistente di 80 Nm;
- la potenza nominale $A'_n$ corrispondente.

In un motore asincrono la coppia è proporzionale al quadrato della tensione, quindi si ha

$$C_{spunto} / C_{pieno\ carico} = (V_{spunto} / V_{pieno\ carico})^2 = (V_{spunto} / 380)^2$$

$$80 / 200 = (V_{spunto} / 380)^2$$

$$V_{spunto} = 380 \text{ sqrt}( 80 / 200 ) = 240 \text{ V}$$

Le correnti sono proporzionali alle tensioni, quindi si ha

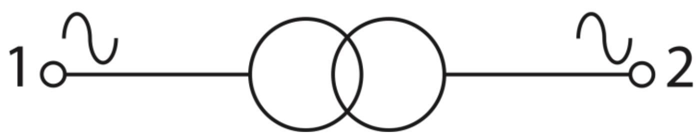
$$I_{240} = I_{380}\ 240 / 380 = 214{,}6 \times 240 / 380 = 135{,}5 \text{ A}$$

Potenza nominale necessaria all'avviamento del motore

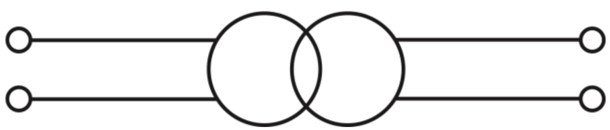
$$A'_n = \text{sqrt}(3) \times 240 \times 135{,}5 = 56{,}7 \text{ kVA}$$

Transformer: static electrical machine operating in AC regime.

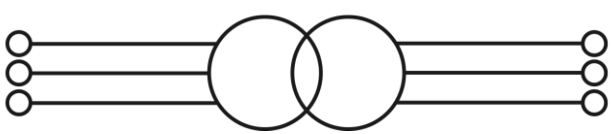
It converts electric power into electric power, by varying the power factors (V,I) and keeping the power (approximately) constant.




1 : primary circuit (power in)
2 : secondary circuit (power out)



Single-phase



Three-phase

## Applications

### Power transformer

- Production systems operate at a reduced voltage
in order to decrease dielectric material size
(medium voltage 15 - 25 kV, 50 Hz)

- Transmission systems operate at a low current
in order to decrease conducting material size
(high voltage 127 – 220 - 380 kV, 50 Hz)
132 – 230 - 400 kV, 50 Hz)

- User systems operate at a low voltage
in order to increase safety
(low voltage 220 – 380 V, 50 Hz, extremely low voltage 12 V DC)

### Signal (and power) transformer

- Adapts the voltage of two systems to be connected
- Adapts the impedance of two systems to be connected
- Couples two systems, keeping them electrically disconnected

Step-up transformer:
increases V (and decreases I)

Step-down transformer:
decreases V (and increases I)

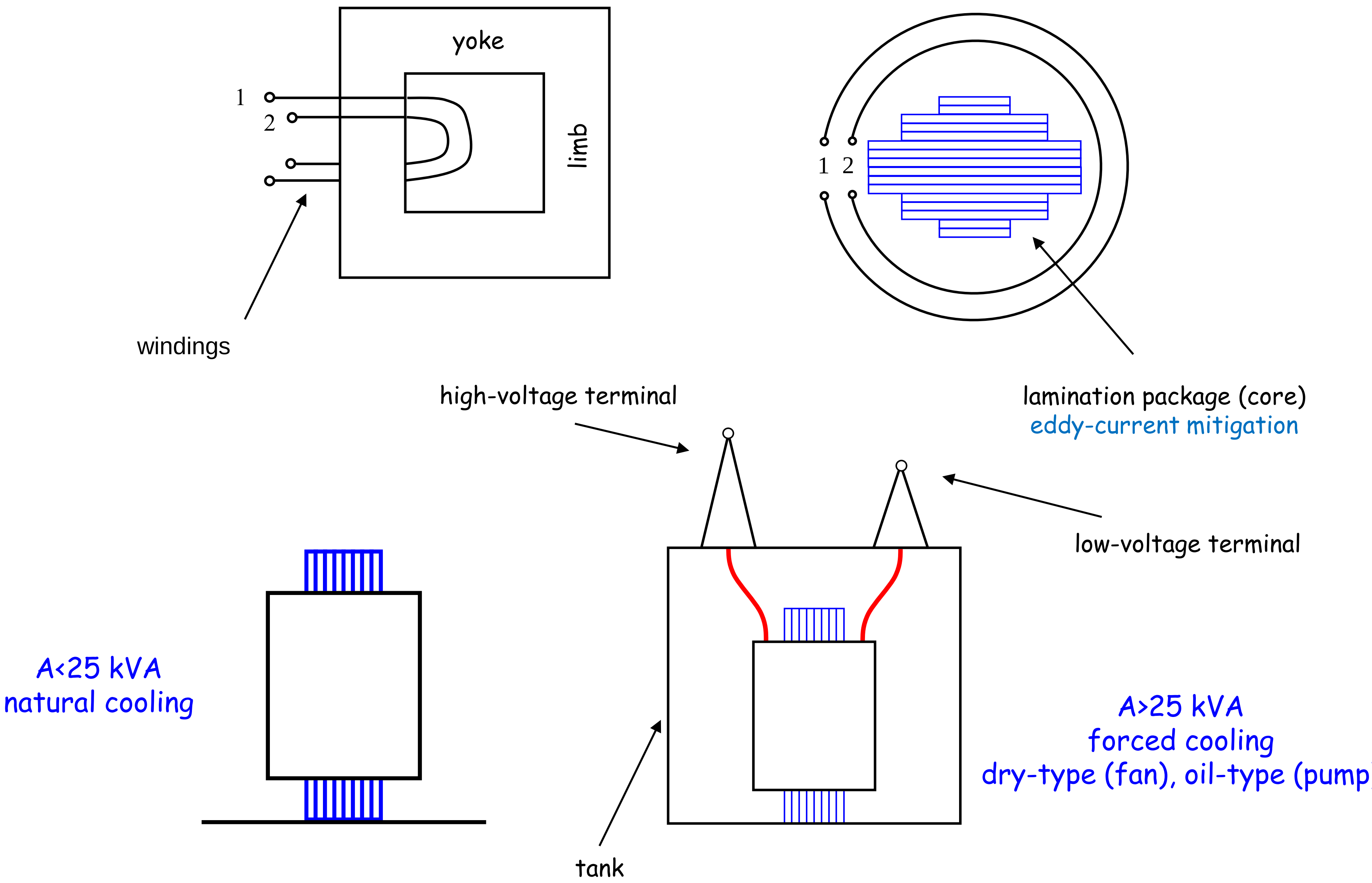
Power range:
from 1 VA to 1000 MVA

Voltage range:
from 1 V to 400 kV

Basically, a transformer is composed of:

- a magnetic circuit;
- (at least) two magnetically-coupled electric circuits.

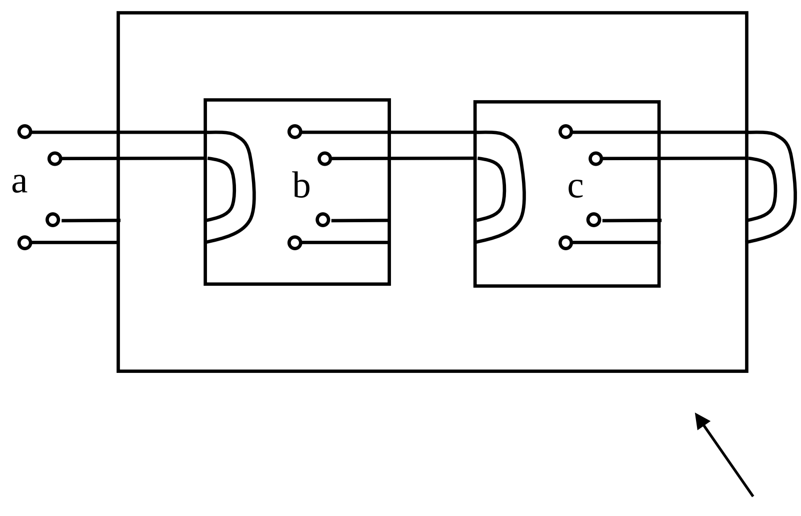
# Single-phase power transformer (25 Hz < f < 400 Hz)



## Three-phase power transformer

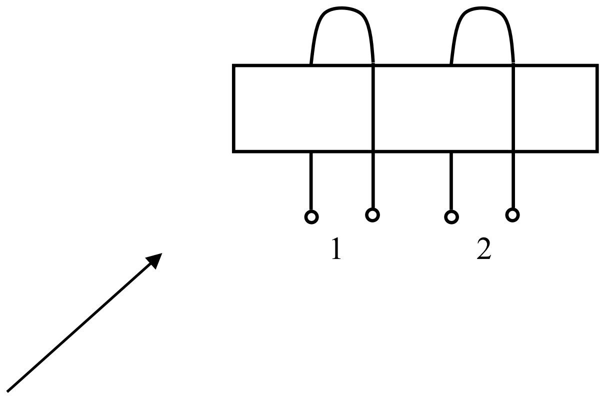
(f=50 Hz) :
three pairs of windings

Three primary windings 1a,1b,1c
and three secondary windings 2a,2b,2c
Used connections:
star-delta, delta-delta, delta-star.




closed magnetic circuit

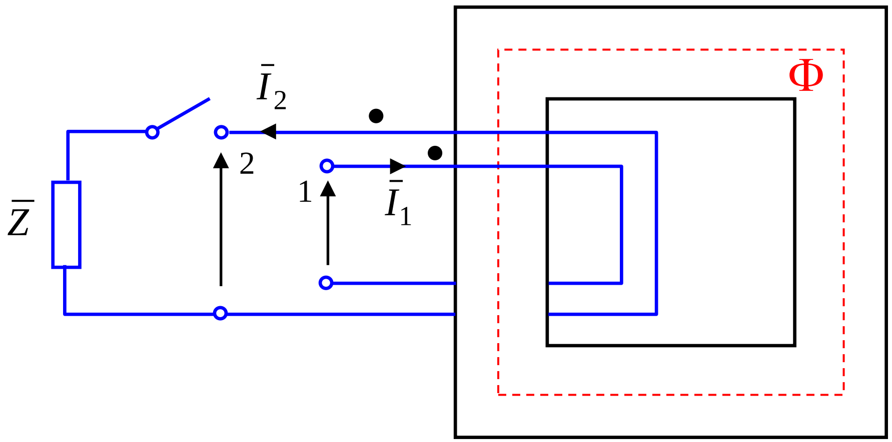
Signal transformer
single phase
20 Hz < f < 20 kHz




open magnetic circuit (f>20 kHz, core made of $Fe_2O_3$)

Low power, high frequency (e.g. telecomunications)

Operating principle (ideal case)
Model assumptions: infinite core permeability,
zero losses in the magnetic circuit and in the two electric circuits.
Moreover: AC regime (phasors).




1: $N_1$ turns

2: $N_2$ turns

Transformer equations

$$\begin{cases} \bar{V}_1 = k\bar{V}_2 \quad , \quad k = \dfrac{N_1}{N_2} \\ \bar{I}_2 = k\bar{I}_1 \\ A_1 = A_2 = V_1 I_1 = V_2 I_2 \end{cases}$$

turn ratio k

# Transformer laws (ideal model) I

No-load operation ($I_2$=0)

Let a sinusoidal voltage source $v_1(t)$ supply winding 1

It happens that an infinitesimal magnetizing current $I_{10}$ flows in winding 1 and gives rise to a magnetomotive force (mmf)

Hopkinson law: magnetic flux

$$N_1 I_{10} = \Theta \Phi$$

(infinitesimal) mmf

(infinitesimal) reluctance

(finite) sinusoidal flux

# Transformer laws (ideal model) II

Flux $\Phi$ links both windings 1 and 2
(flux linkage: $\Phi_{c1} = N_1\Phi$, $\Phi_{c2} = N_2\Phi$)

Faraday-Lenz law: induced emf

$$e_1 = -N_1 \frac{d\Phi}{dt} \qquad e_2 = -N_2 \frac{d\Phi}{dt}$$

KVL $\quad v_1+e_1=0$, $v_2+e_2=0$ → $v_1=-e_1$, $v_2=-e_2$

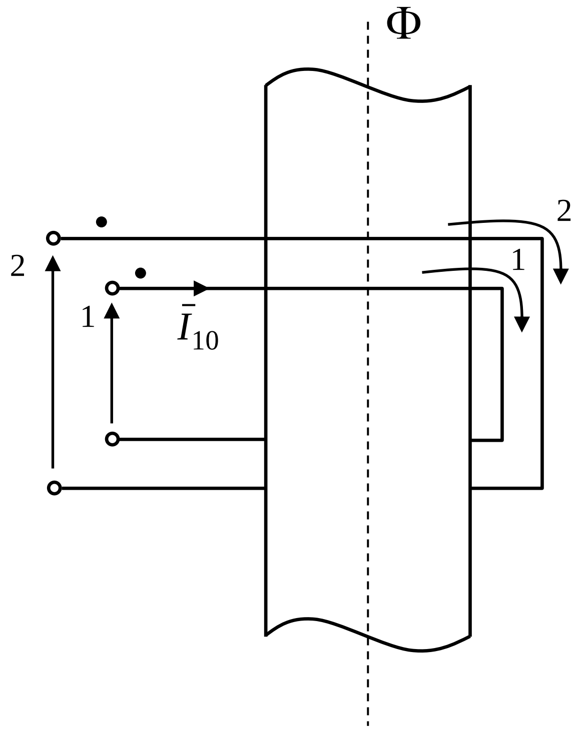
In phasor notation: $\quad \overline{V}_1 = j\omega N_1 \overline{\Phi} \qquad \overline{V}_2 = j\omega N_2 \overline{\Phi}$

In terms of RMS values, it turns out to be:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = k$$

transformer voltage law

Primary circuit: supplied by $V_1$
Secondary circuit: open circuited




Kirchhoff voltage law

$v_1+e_1=0$ , $v_2+e_2=0$

$$\overline{V}_1+\overline{E}_1=0$$

$$\overline{V}_2+\overline{E}_2=0$$

$I_{10}$ magnetizing current

Terminal marking: currents entering the marked terminals originate like fluxes in the magnetic core.

## Transformer laws (ideal model) III

On-load operation ($I_2 \neq 0$)

Current $I_2$ is delivered by winding 2 to load Z

mmf $N_2I_2$ originates a secondary flux $\Phi_2$
opposing the primary flux $\Phi_1 = \Phi_M$ , but

$$\Phi_M = \frac{V_1}{4.44\, N_1\, f}$$

cannot decrease due to the applied primary voltage $V_1$

## Transformer laws **(ideal model) IV**

Consequently, due to the flux conservation law, voltage source $V_1$ is forced to deliver an additional current $I_1$ to winding 1 in order to compensate the flux mismatch:

$N_1 I_1 - N_2 I_2 = N_1 I_{10} = 0$

It turns out to be:

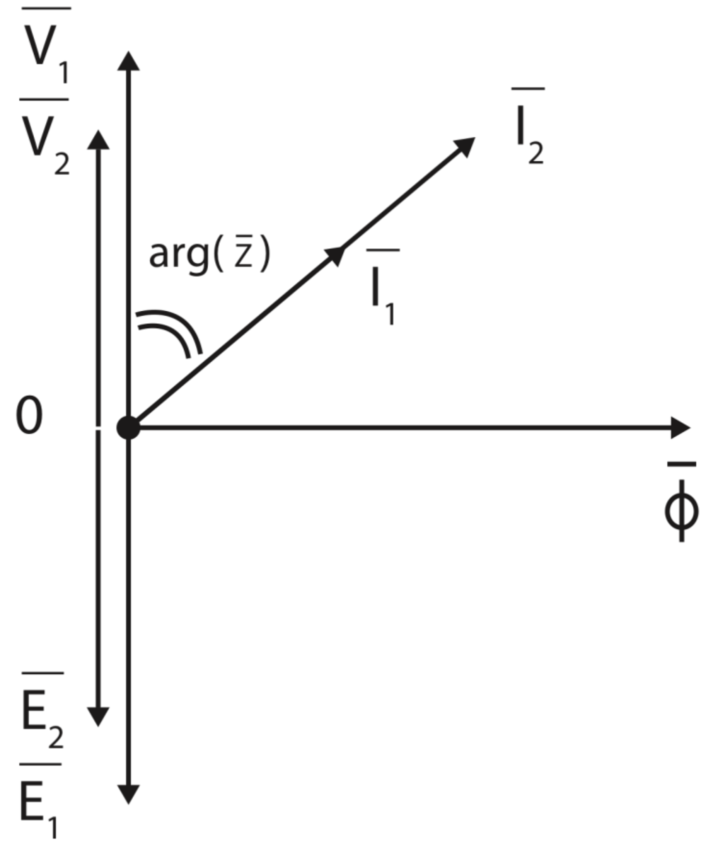
$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{k}$$ transformer current law

Finally, the complex power conservation holds:

$$V_1 I_1 = V_2 I_2$$

Vector diagram

(Kapp diagram)




A resistive-inductive load is assumed

# A more realistic model - 1

In a real transformer, power losses are not zero:

$P_0=GV^2$ active power in the magnetic core (eddy current and hysteresis)
$P_c=RI^2$ active power in the electric circuits due to the Joule effect

Moreover, the finite magnetic permeability of the core causes a non-zero magnetizing power:

$Q_0=BV^2$ reactive power in the magnetic circuit of main flux (inside the core)
$Q_C=XI^2$ reactive power in the magnetic circuit of stray flux (outside the core)

# A more realistic model - 2

$P_0$ and $Q_0$ dominate in the no-load operation:

measured in the open-circuit test

$P_c$ and $Q_c$ dominate in the on-load operation:

measured in the short-circuit test

Short circuit voltage: reduced value of primary voltage $V_{1c}$ such that the short-circuit secondary current $I_{2c}$ equals the maximum load current $I_{2n}$ (i.e. $V_2 = 0$ , $I_2 = I_{2n}$)
Normally, $V_{1c} = 0.04 * V_{1n}$ with $V_{1n}$ full voltage value holds

# A more realistic model - 3

As a consequence, in a real transformer:

$V_2$ is different from $E_2$ due to voltage drops caused by R and X of winding 2 when on load

$V_1$ is different from $kV_2$ due to on-load voltage drops

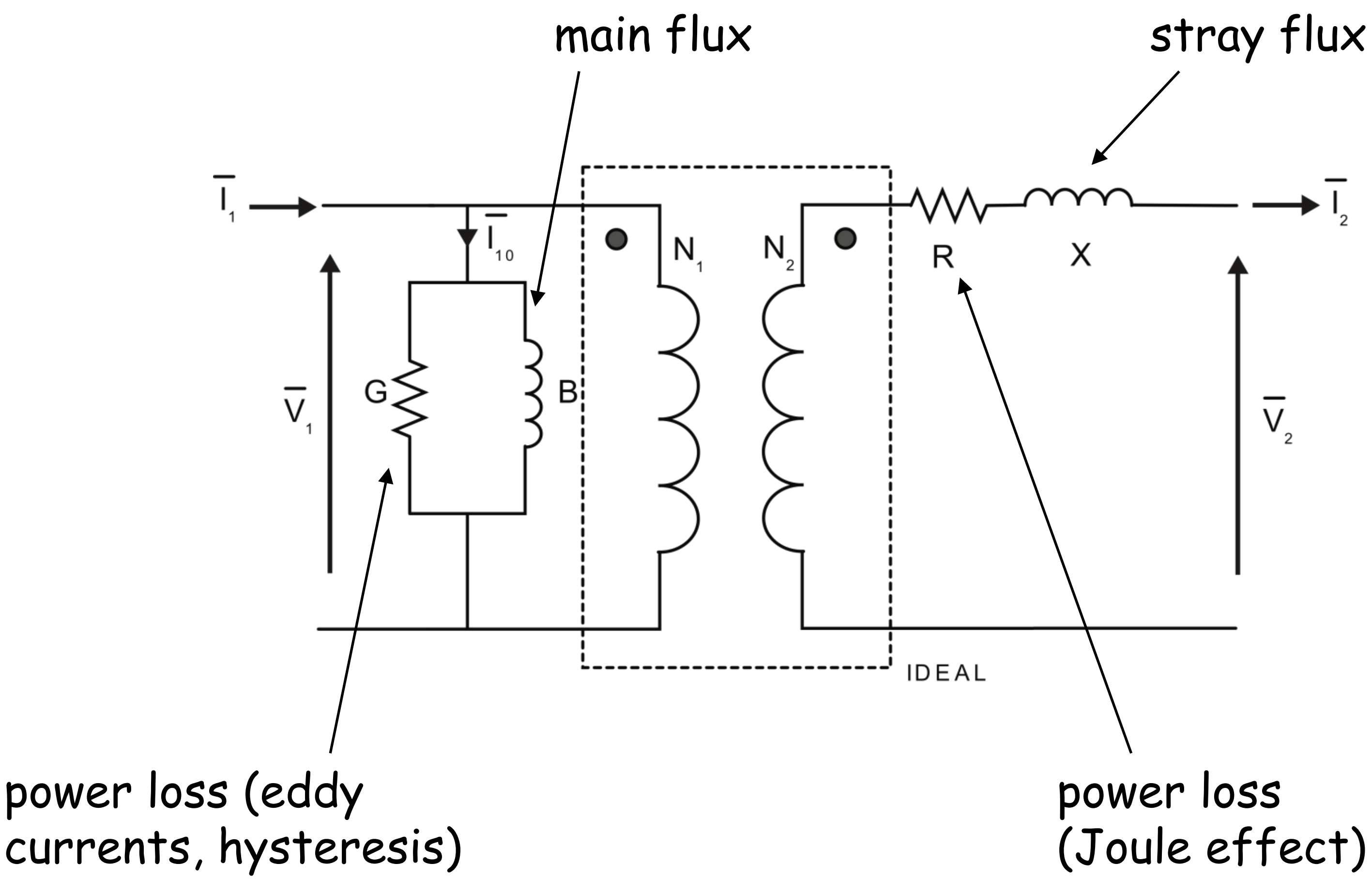
$I_1$ is different from $I_2/k$ due to no-load primary current

$\Phi$ is not constant with respect to $E_1$ because $E_1$ varies with $I_1$ for a given $V_1$

$A_1$ is different from $A_2$ due to active and reactive power losses

Transformer equations based on the ideal model are valid as a first approximation only

# A more realistic model - 3



# A more realistic model - 4

$$\begin{cases} \bar{V}_2 = \frac{1}{k}\bar{V}_1 - \bar{Z}\bar{I}_2 \\ \bar{I}_1 = \frac{1}{k}\bar{I}_2 + \bar{Y}\bar{V}_1 \\ P_1 = P_2 + RI_2^2 + GV_1^2 \\ Q_1 = Q_2 + XI_2^2 + BV_1^2 \end{cases}$$

$$\bar{Z} = R + jX$$

$$\bar{Y} = G + jB$$

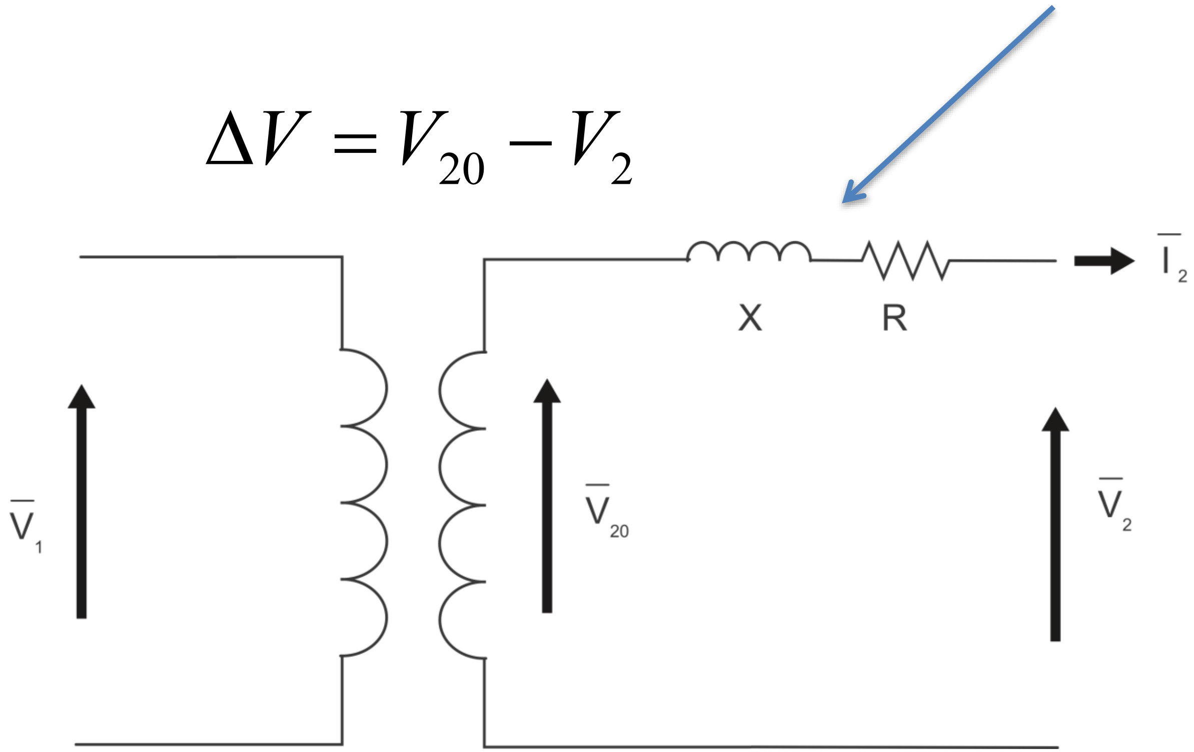
Primary winding admittance

Secondary winding impedance

# Voltage drop - 1

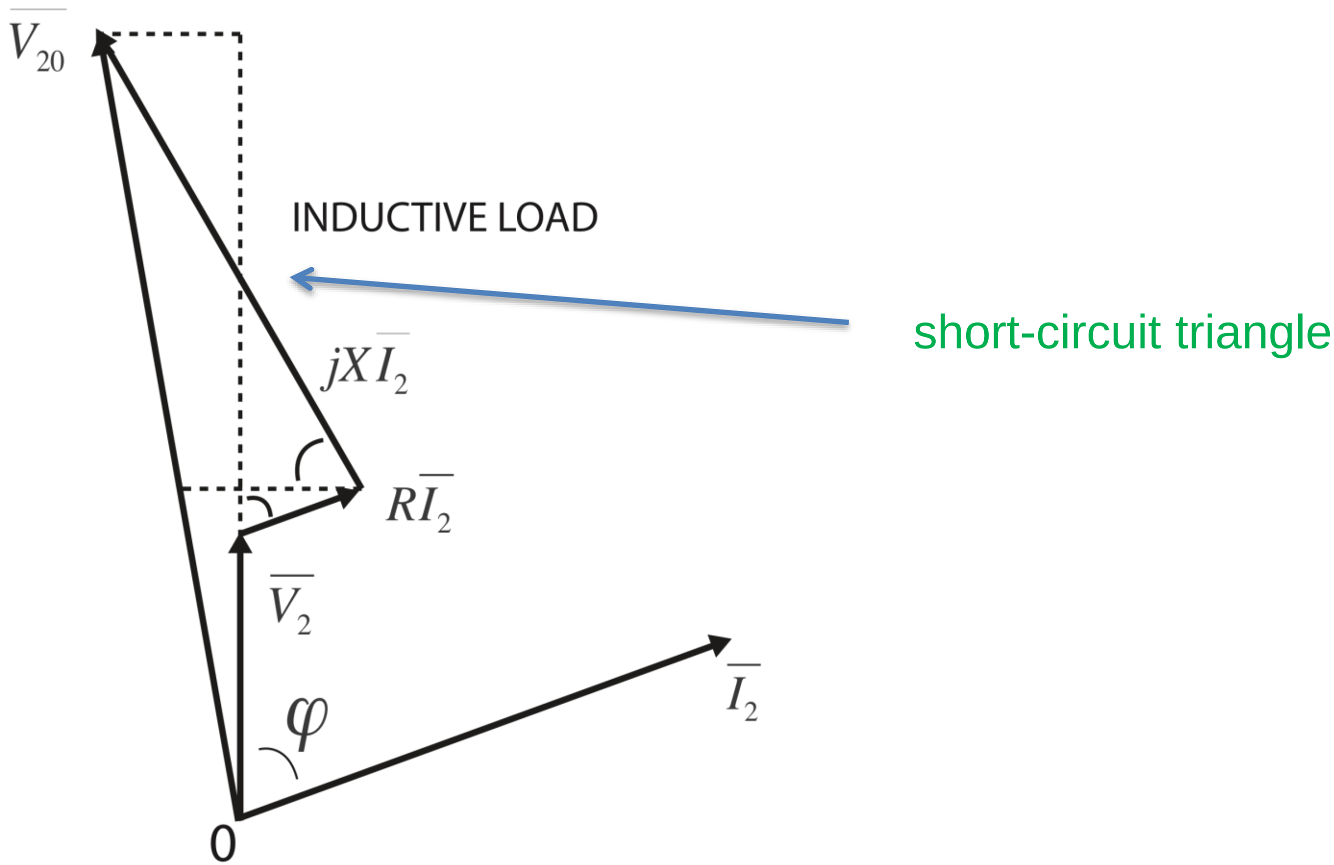
short-circuit impedance

$$\Delta V = V_{20} - V_2$$




$$\frac{1}{k}\bar{V}_1 = \bar{V}_{20} = \bar{V}_2 + (R + jX)\bar{I}_2 = V_2 + (R + jX)(I_2 \cos\varphi + jI_2 \sin\varphi)$$

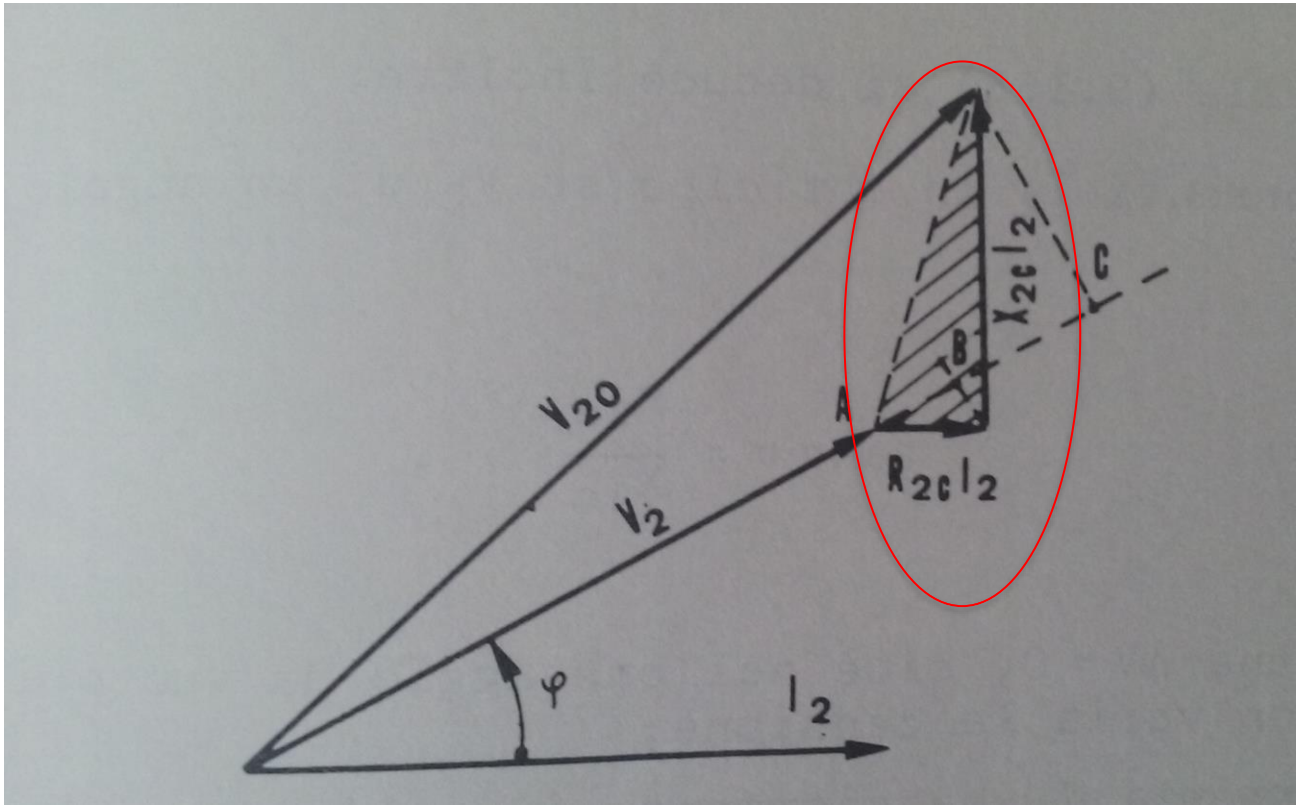
# Voltage drop - 2




$$\Delta V \equiv V_{20} - V_2 \cong RI_2 \cos\varphi + XI_2 \sin\varphi$$

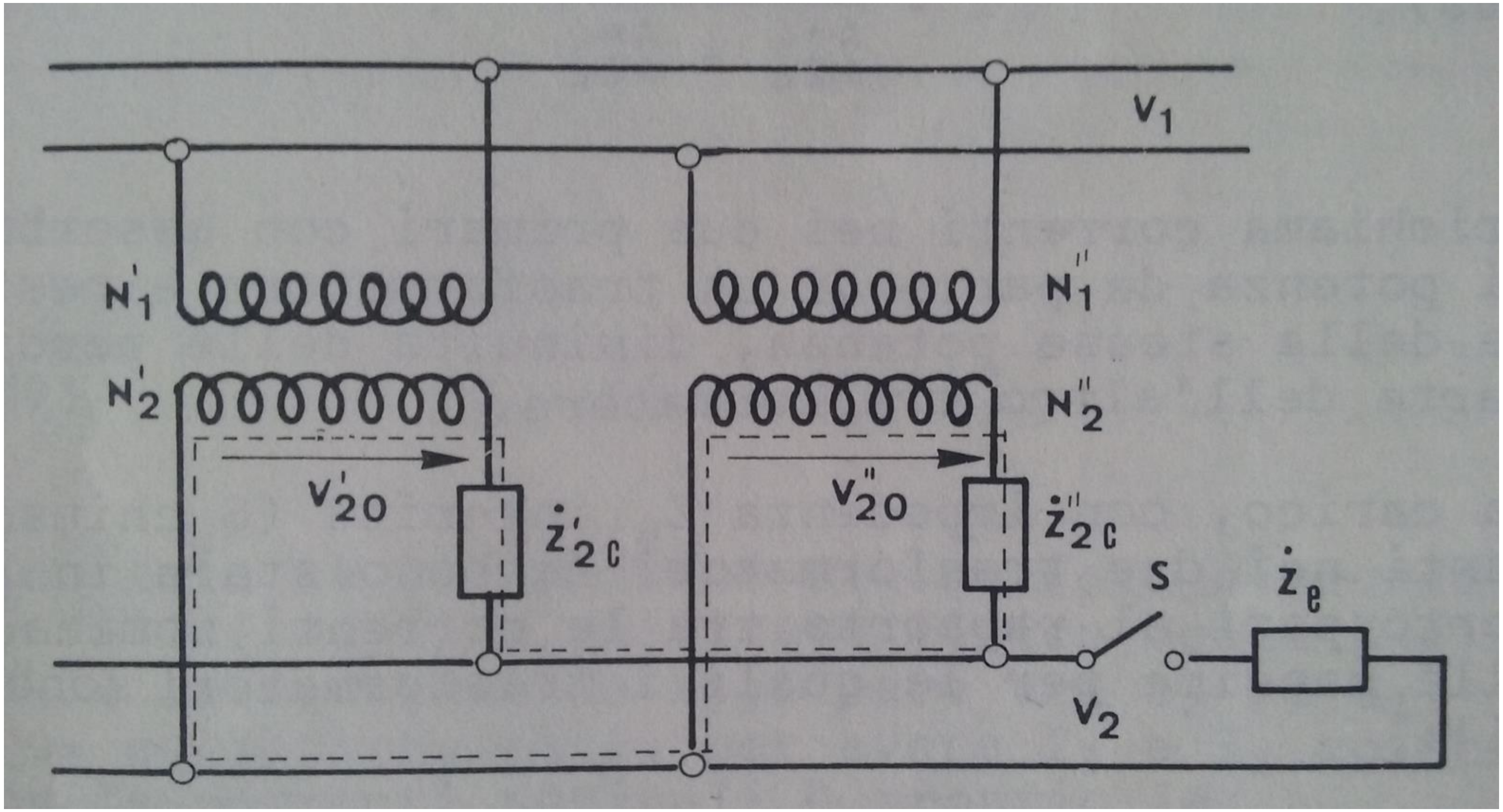
# Short circuit triangle

If $V_2=0$ and $I_2=I_{2n}$
then $V_{20}$ is the short-circuit secondary voltage
It corresponds to the triangle hypotenuse



# Parallel connection of two transformers

Useful for increasing the power delivered to a load




Possible if both transformers exhibit:

- same turn ratio ( => zero no-load currents)
- same short-circuit voltage ( => balanced on-load currents, such that $I_2'/I_2''=I_{2n}'/I_{2n}''$)

$$I_2' / I_2'' = I_{2n}' / I_{2n}''$$

$$V_2' = V_{20}' - Z_{2c}' I_2' , \quad V_2'' = V_{20}'' - Z_{2c}'' I_2''$$

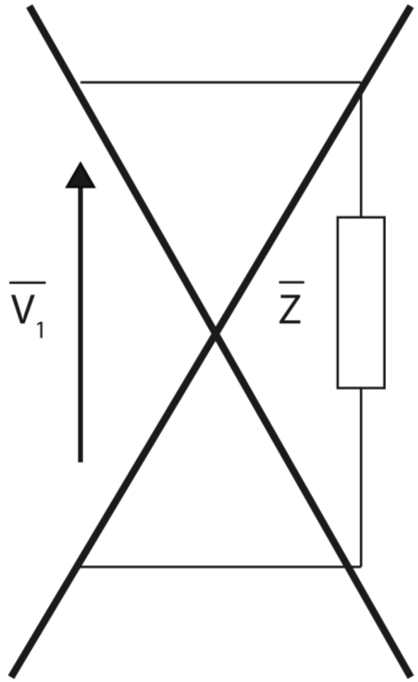
$$V_2' = V_2'' \longrightarrow Z_{2c}' I_2' = Z_{2c}'' I_2''$$

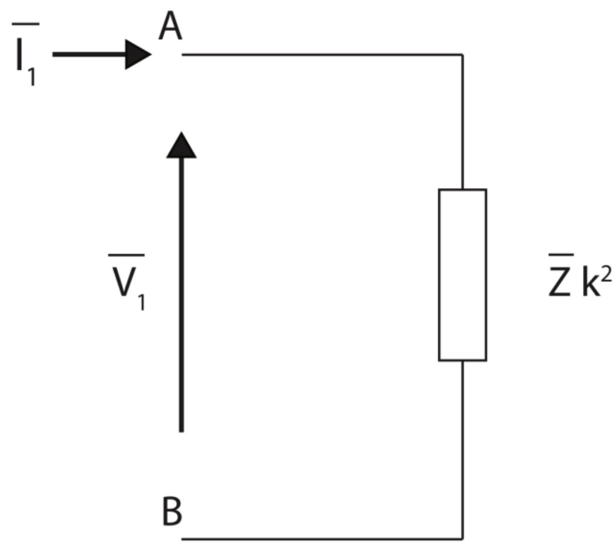
$$Z_{2c}' I_{2n}' = Z_{2c}'' I_{2n}''$$

$$V_{2c}' = V_{2c}''$$

# Impedance adaption - 1

Problem: how to modify the value of a load impedance Z to be connected at a terminal pair A-B without changing the impedance?



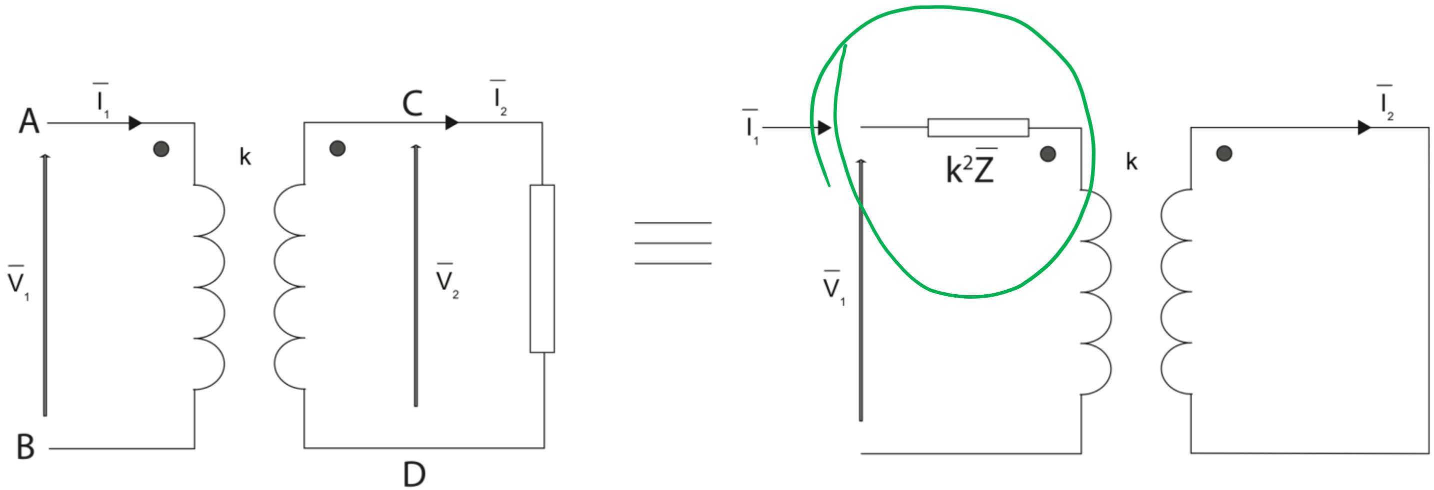




$$\bar{I}_1 = \frac{\overline{V}_1}{\overline{Z}}$$

$$\bar{I}_1 = \frac{\overline{V}_1}{k^2\,\overline{Z}} \qquad k \neq 1$$

# Impedance adaption - 2

Solution: insert a transformer between source and load




$$\bar{I}_1 = \bar{I}_2 \frac{1}{k} = \frac{\bar{V}_2}{\bar{Z}} \frac{1}{k} = \frac{\bar{V}_1}{k} \frac{1}{\bar{Z}} \frac{1}{k} = \frac{\bar{V}_1}{k^2 \bar{Z}}$$

Equivalence: load $k^2Z$ series connected to the source, with short-circuited secondary winding

# Trasformatore per adattamento di impedenza

Un altoparlante con resistenza $R_a$ = 4 ohm è alimentato da un amplificatore, rappresentabile mediante un generatore reale di tensione PAS (E = 10 V) con resistenza serie $R_s$ = 1 ohm.

Si calcoli la potenza fornita all'altoparlante quando è collegato:

- direttamente all'amplificatore;
- al secondario di un trasformatore di rapporto k = 1/2 con il primario alimentato dall'amplificatore.

Caso 1

$P_a = V_a I_a = E R_a /(R_s+R_a) * E/(R_s+R_a) = 100\ 4/5^2 = 16\ W$

Caso 2

Resistenza ai morsetti del primario $R_a' = k^2 R_a = 4/2^2 = 1\ ohm = R_s$

Tensione e potenza di $R_a'$

$V_a' = E R_a'/(R_s+R_a') = 10 / 2 = 5\ V$

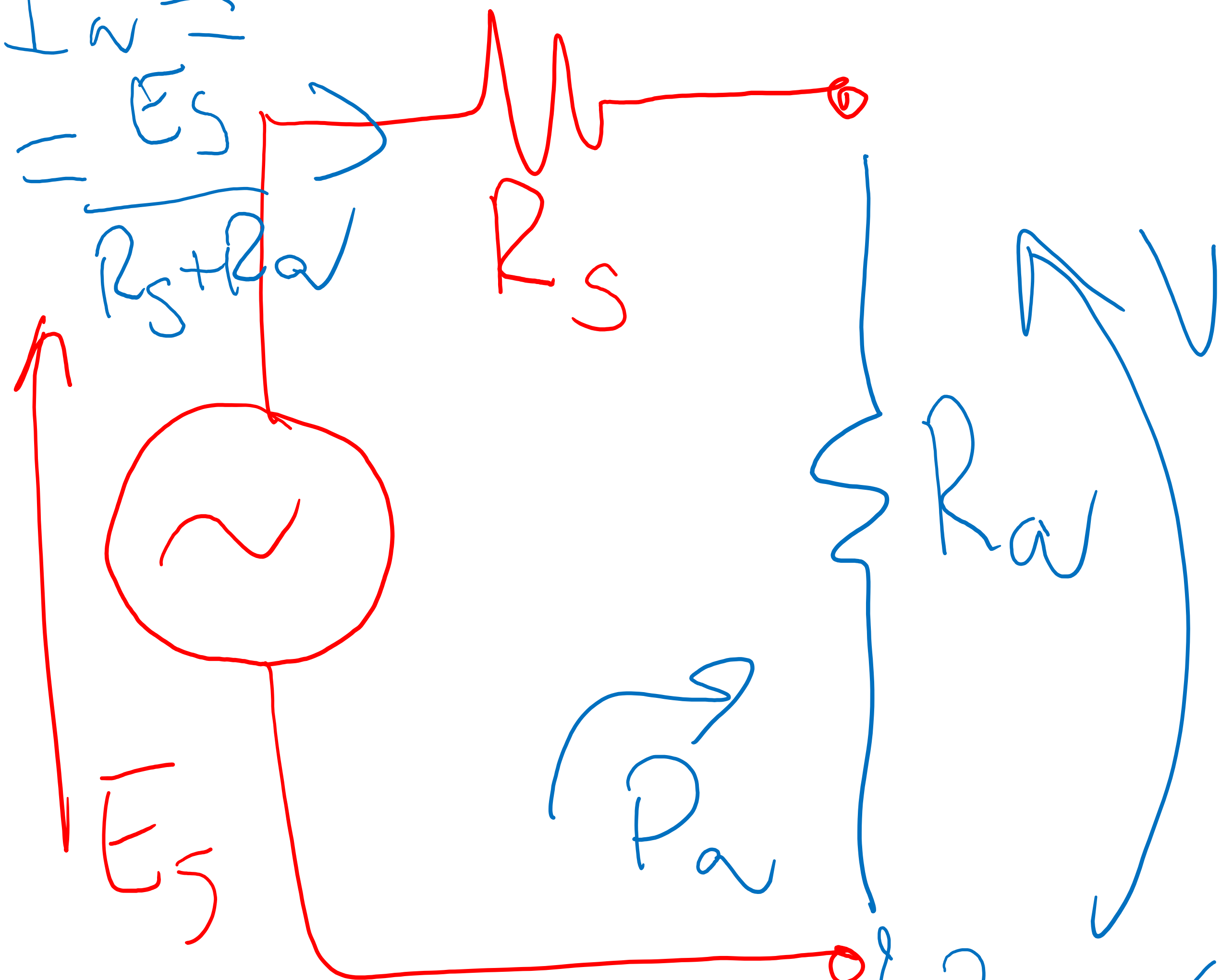
$P_a' = V_a'\ E/(R_s+R_a') = 50 / 2 = 25\ W$

Tensione e potenza di $R_a$

$V_a = V_a' / k = 10\ V$

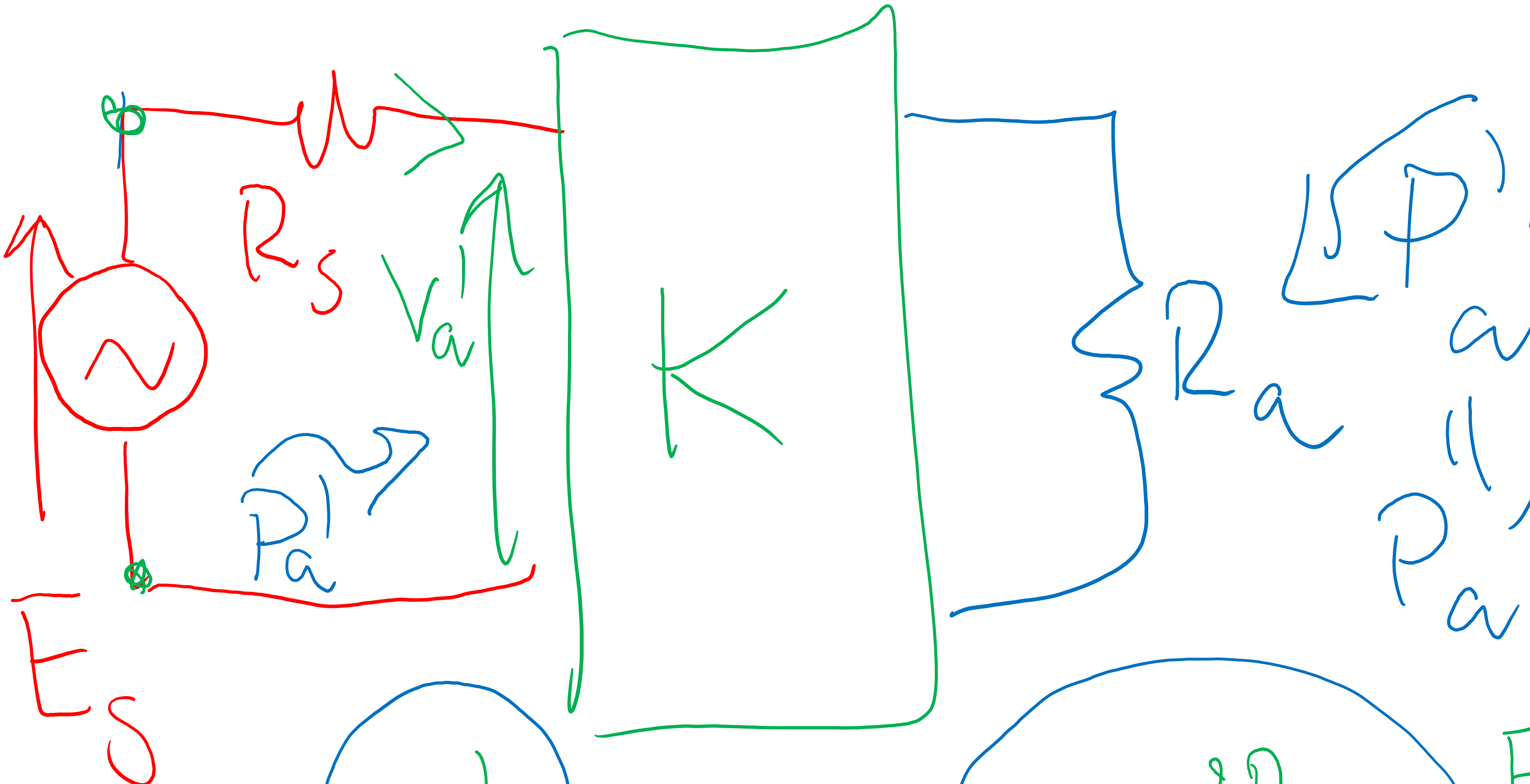
$P_a = V_a^2/R_a = 10^2/4 = 25\ W > 16\ W$

$$I_a = \frac{E_s}{R_s+R_a}$$




$$V_a = \frac{E_s R_a}{R_s+R_a}$$

$$P_a = E^2 R_a / (R_s+R_a)^2$$




$$P_a' = \frac{V_a' \cdot E_s}{R_s + K^2 R_a} = \frac{E_s \cdot K^2 R_a}{R_s + K^2 R_a} \cdot \frac{E_s}{R_s + K^2 R_a}$$

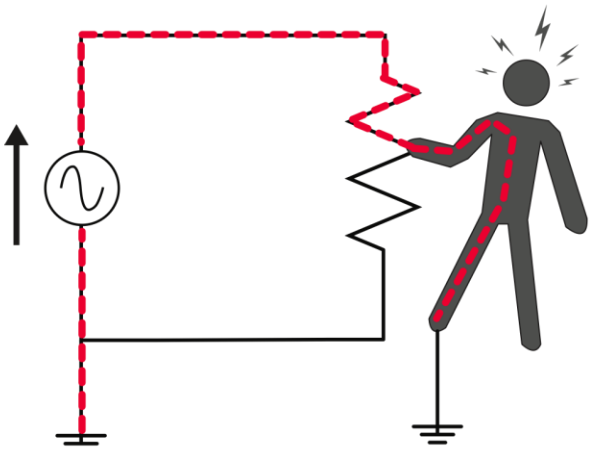
$$P_a'' = P_a' = \frac{E_s^2 \, K^2 R_a}{(R_s + K^2 R_a)^2}$$


$$K^2 R_a = R_s$$

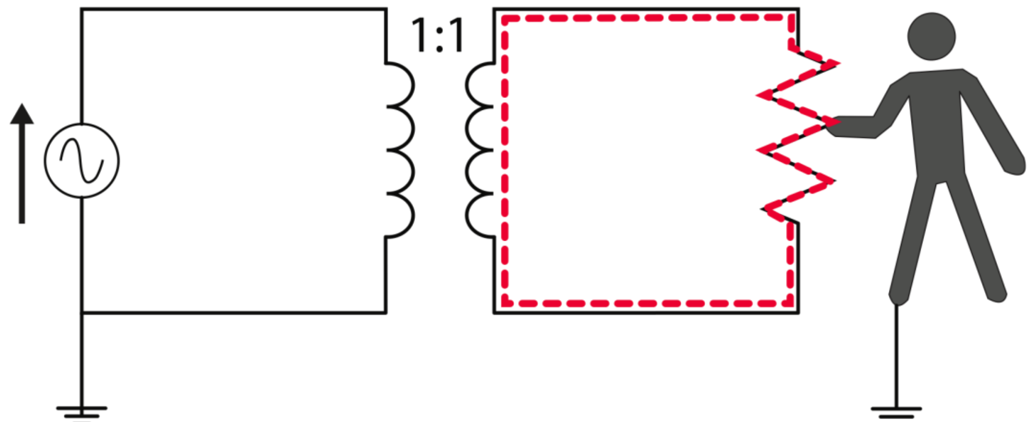
$$K = \sqrt{\frac{R_s}{R_a}} = \frac{1}{2}$$

# Insulation transformer - 1

## Safety



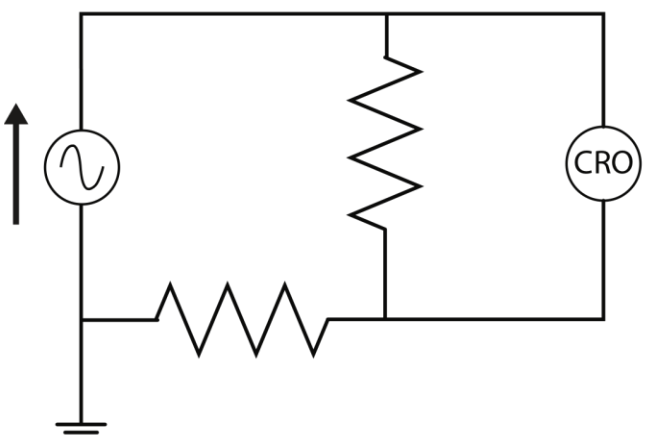
The risk of injury is highest, because the human body closes a circuit towards ground.

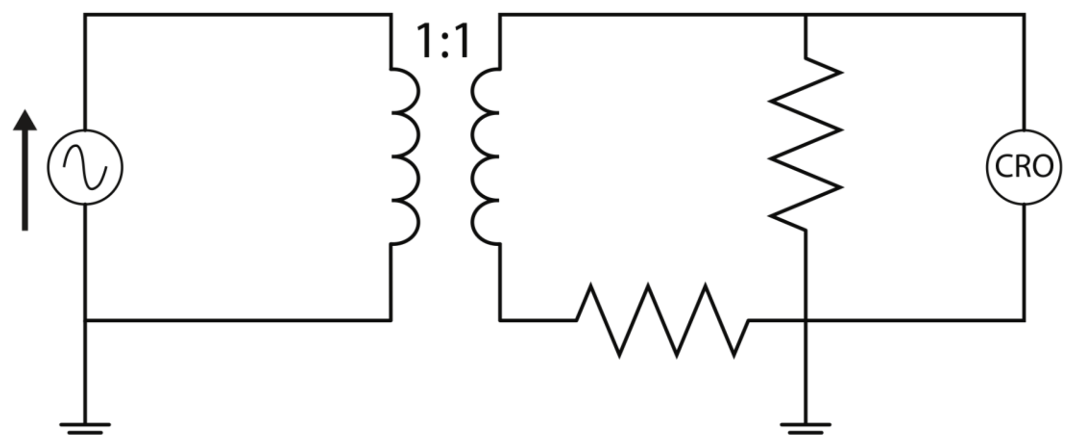



In this case the human body is safe: in fact, it does not close any circuit and, therefore, no current can pass through it.

# Insulation transformer - 2

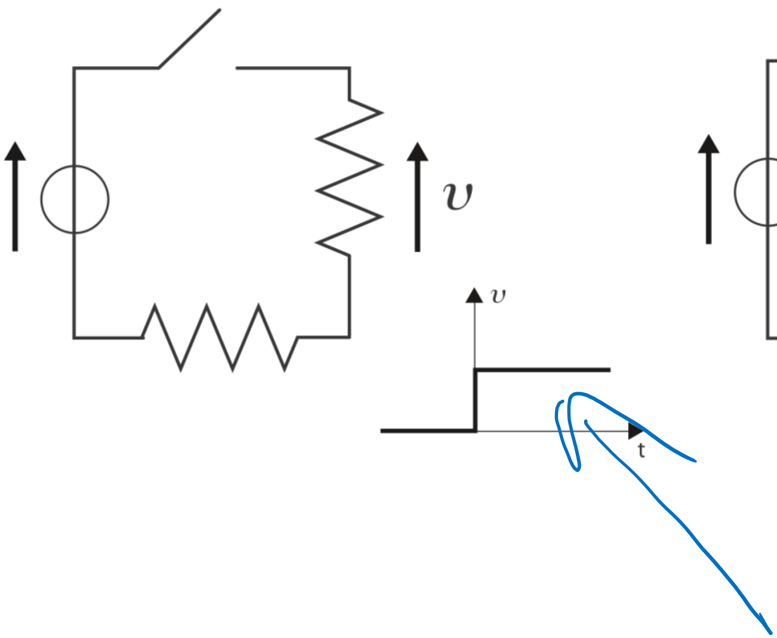
## Measurement

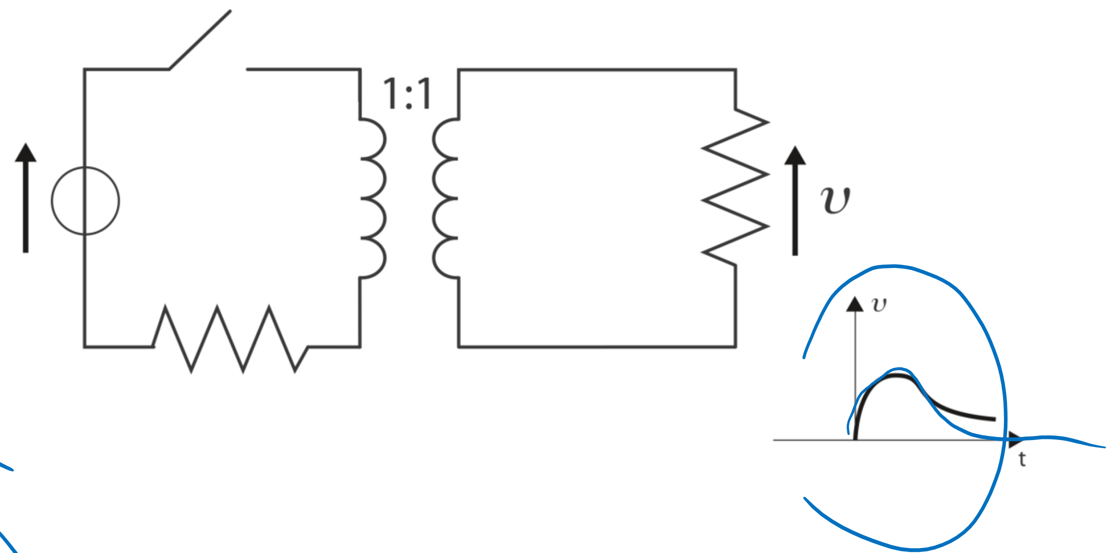


# Insulation transformer - 3

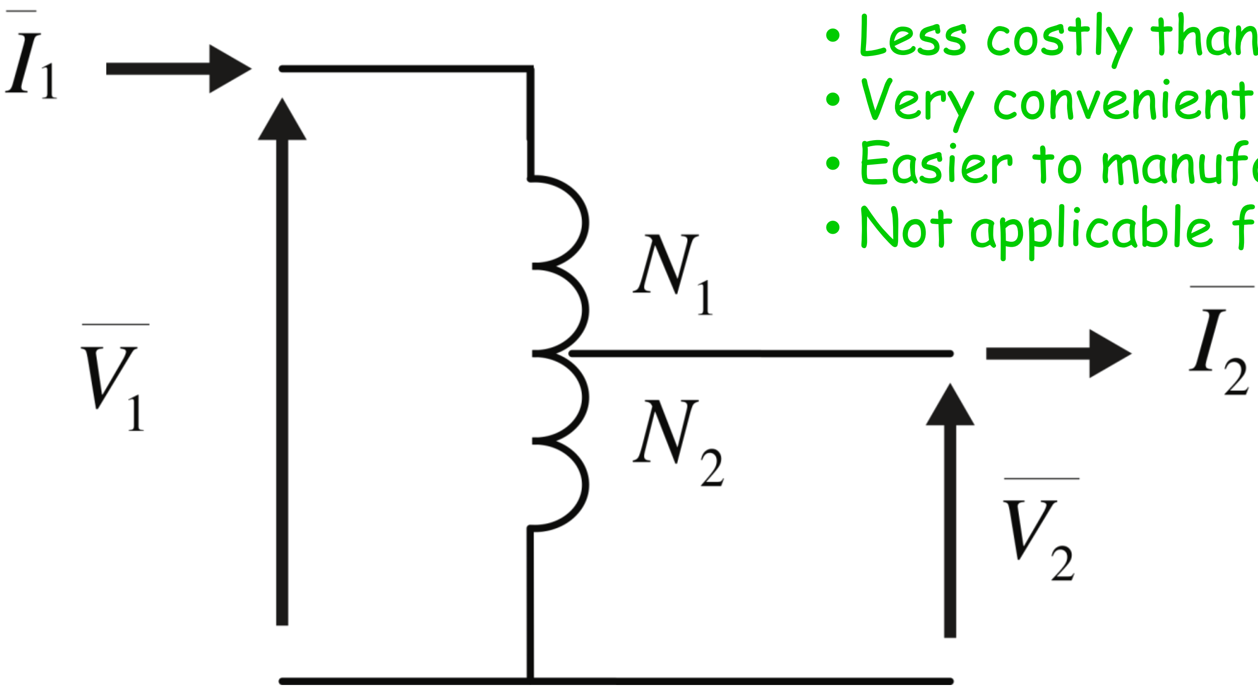
## Damping



# Auto-transformer

- Less costly than the ordinary one
- Very convenient when $N_1 \sim N_2$
- Easier to manufacture
- Not applicable for safety




$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{N_2 + N_1}{N_2} = k \qquad \frac{\bar{I}_1}{\bar{I}_2} = \frac{N_2}{N_2 + N_1} = \frac{1}{k}$$

$$A_1 = V_1 I_1 = V_2 I_2 = A_2 = A$$

Potenze apparenti

$$A_1 = V_1 I_1 = V_2 I_2 = A_2 = A$$

Potenza di dimensionamento

$$A_d = (V_1 - V_2) I_1 = (I_2 - I_1) V_2 \longrightarrow \frac{A_d}{A} < 1$$

Infatti si ha:

$(V_1 – V_2)\ I_1 = (V_1 – V_1/k)\ I_1 = (k-1)/k\ V_1 I_1 < A$

ed anche

$(I_2 – I_1)\ V_2 = (kI_1 – I_1)\ V_2 = (k-1)\ I_1 V_1/k = (k-1)/k\ V_1 I_1 < A$

Three-phase
power transformer
(f=50 Hz) :
three pairs of windings

Three primary windings 1a,1b,1c
and three secondary windings 2a,2b,2c
Used connections:
star-delta, delta-delta, delta-star.